Lecture on rigid dynamics

Sri.S. N. Mishra

RIGID DYNAMICS

Planar Kinetics of a Rigid Body: Force and Acceleration

Objective

- Moment of Inertia of a body
- Parallel Axis Theorem
- Radius of Gyration
- Moment of Inertia of Composite Bodies

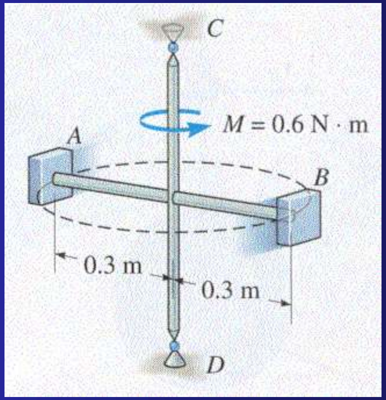
Moment and Angular Acceleration

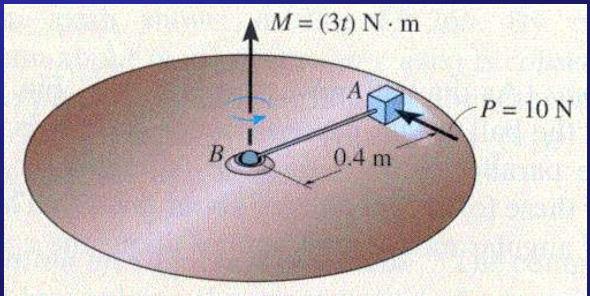
- When M ≠ 0, rigid body experiences angular acceleration
- Relation between M and α is analogous to relation between F and a

$$F = ma, \qquad M = I\alpha$$

Mass = Resistance

Moment of Inertia



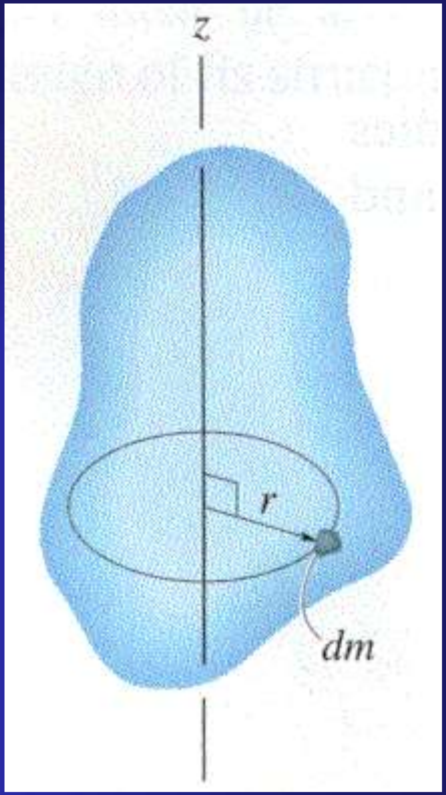


Moment of Inertia

- This mass analog is called the *moment of inertia*, I, of the object

$$I = \int_m r^2 dm$$

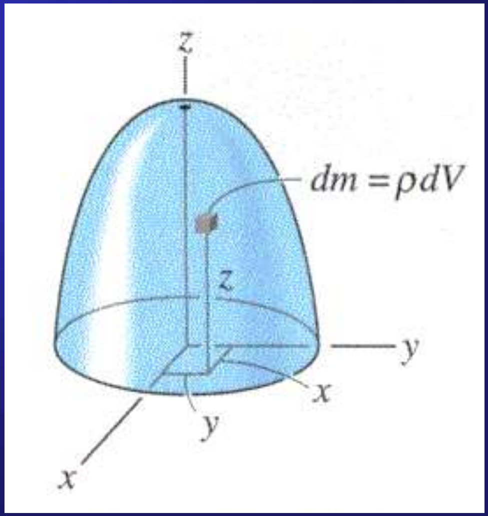
  - r = moment arm
  - SI units are kg m^2



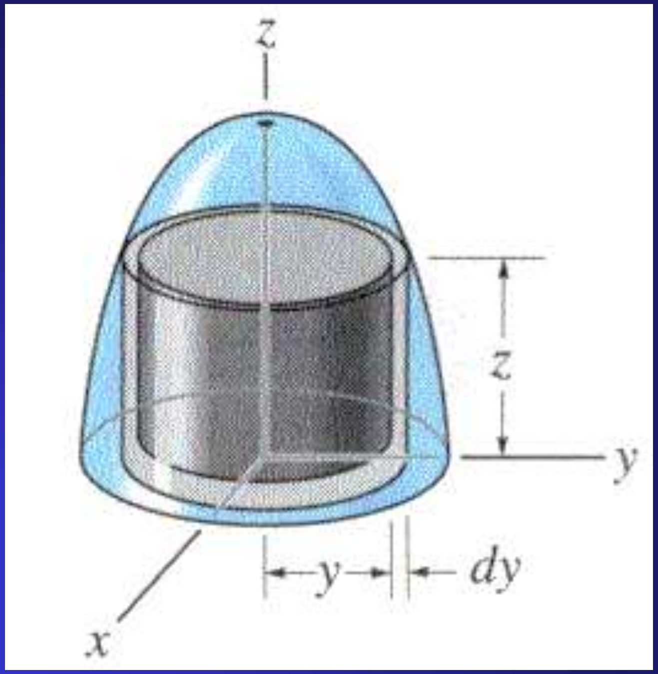
Using $dm = \rho\, dV$, where ρ is the volume density :

$$I = \int \rho\, r^2 dV$$

$$I = \rho \iiint r^2\, dx\, dy\, dz$$

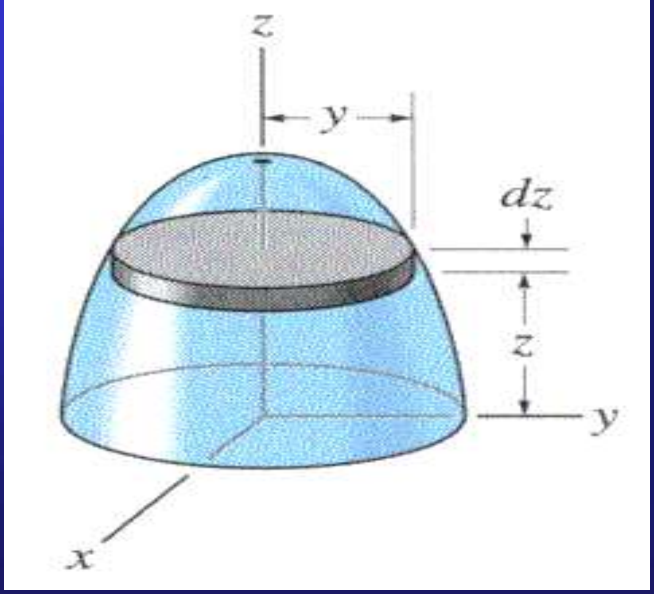


Shell Element



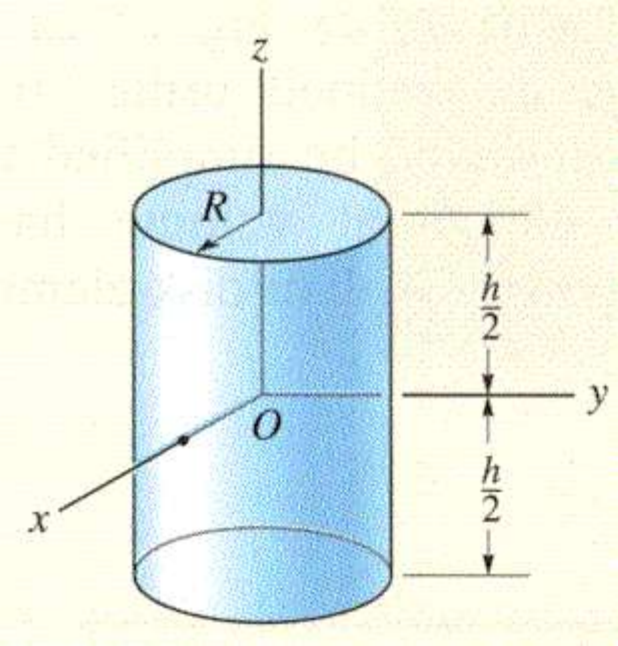
$$dV = (2\pi\, y)\, z\, dy$$

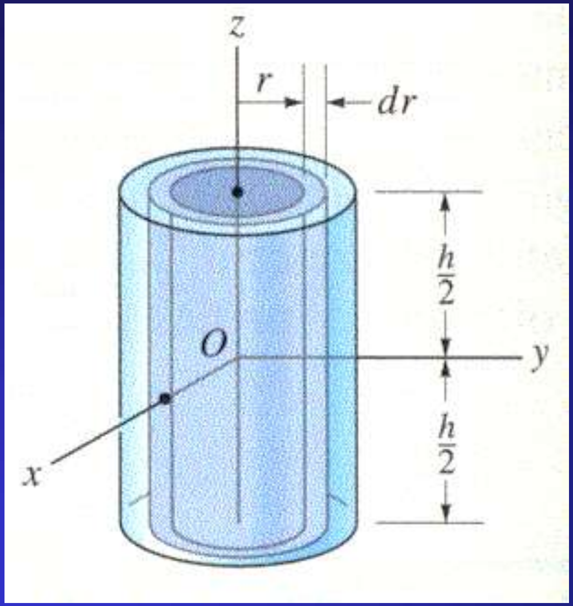
Disk Element



$$dV = (\pi\, y^2)dz$$

Example 17-1



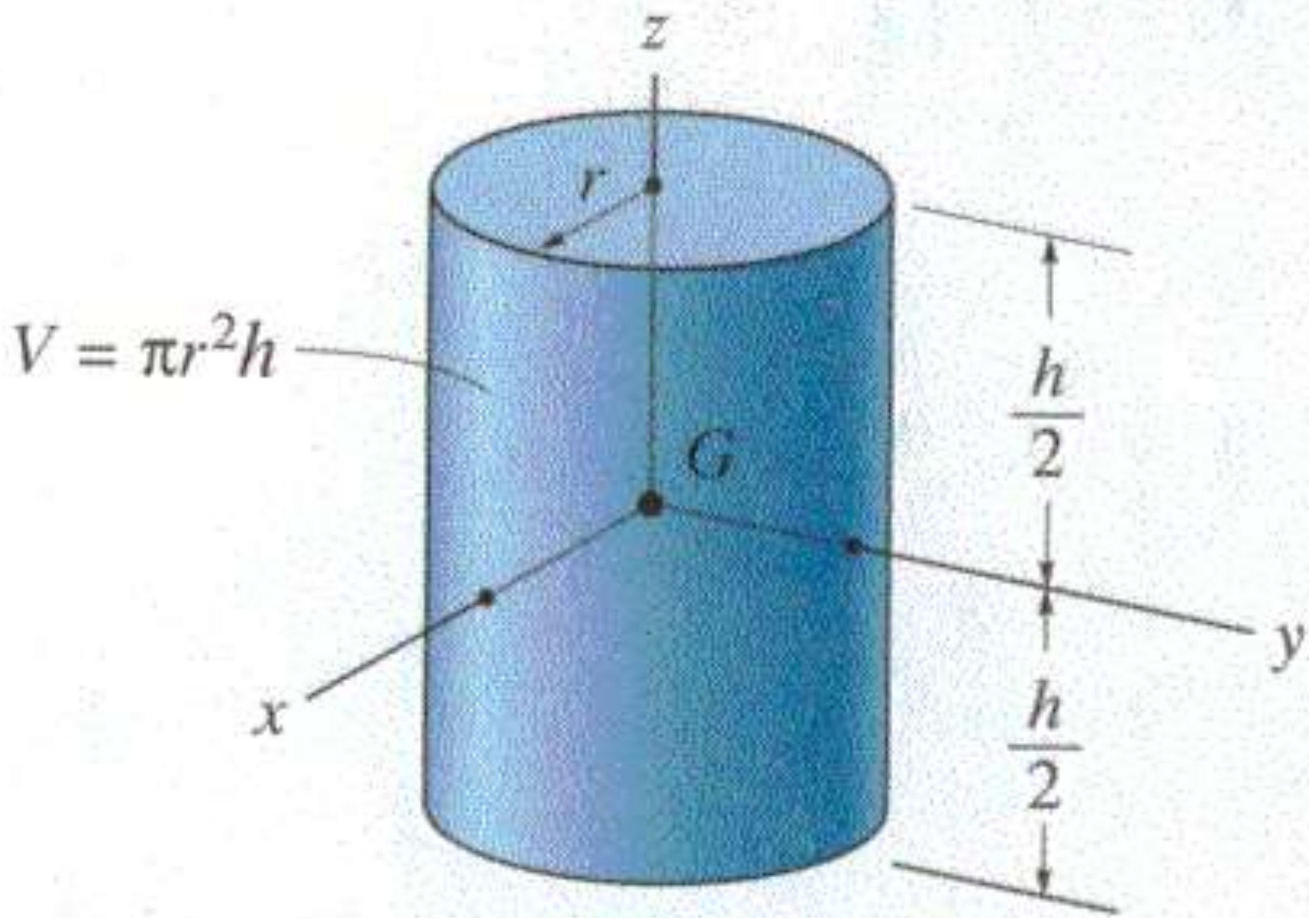


$$dm = \rho\, dV = \rho\,(2\pi\, r\, dr\; h)$$

$$I = \int_m r^2 dm = \rho\, 2\pi\, h \int_0^R r^3\, dr = \frac{\rho\pi}{2} R^4 h = \frac{1}{2} R^2 (\rho\pi R^2 h)$$

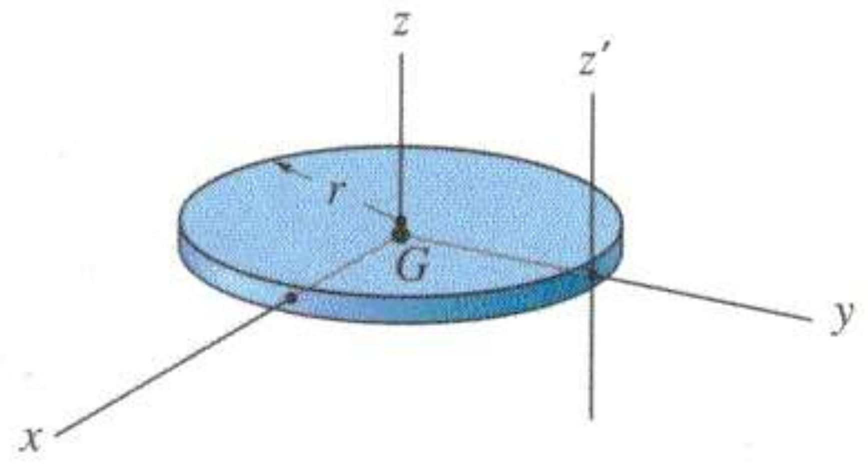
$$m = \rho\, \pi\, R^2 h$$

$$\boxed{I_z = \frac{1}{2} m\, R^2}$$



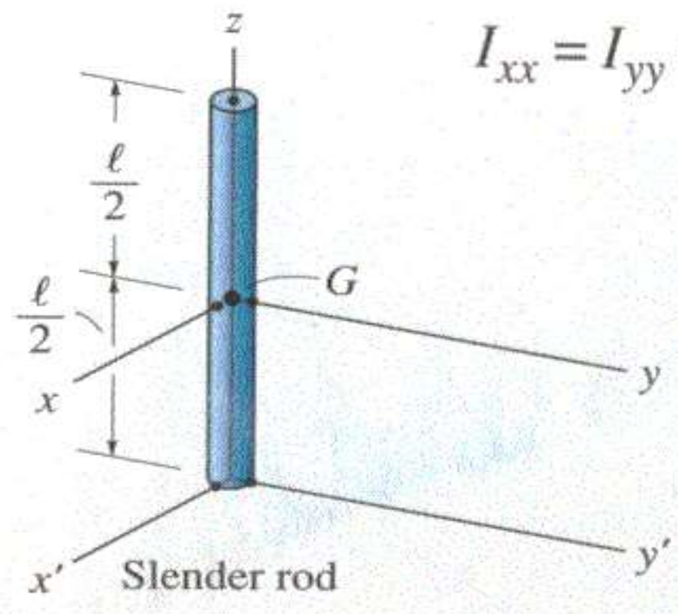
Cylinder

$$I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2}mr^2$$



Thin circular disk

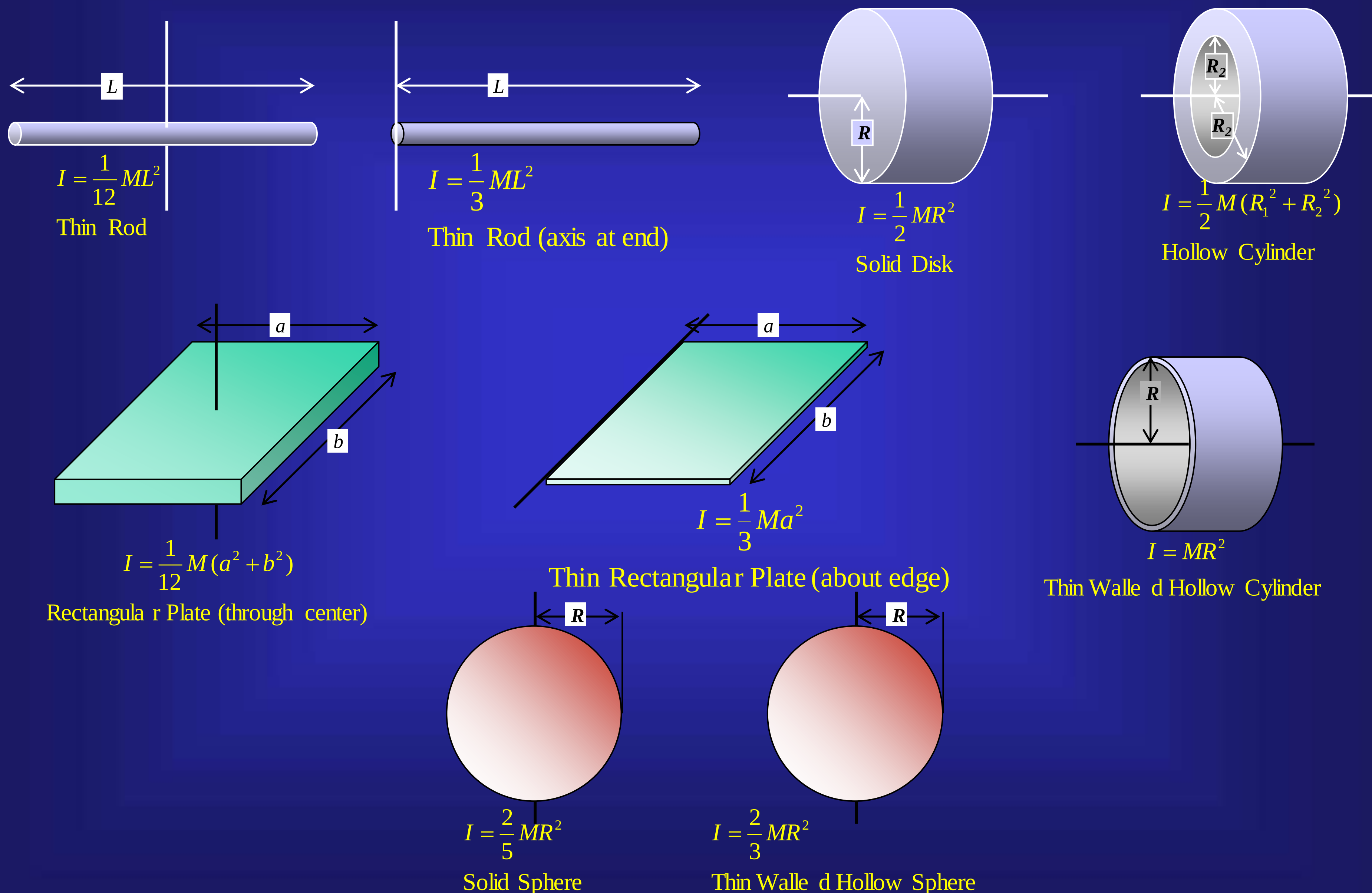
$$I_{xx} = I_{yy} = \tfrac{1}{4}mr^2 \quad I_{zz} = \tfrac{1}{2}mr^2 \quad I_{z'z'} = \tfrac{3}{2}mr^2$$



Slender rod

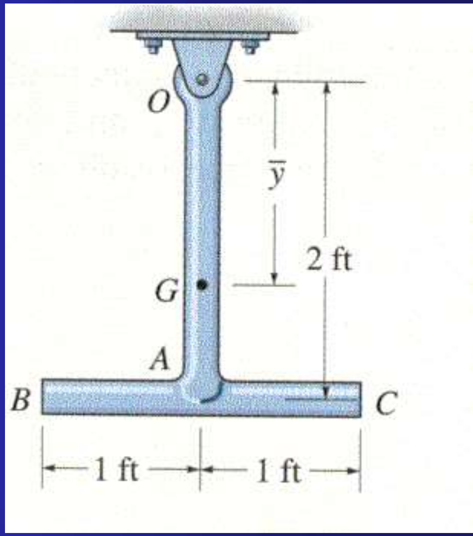
$$I_{xx} = I_{yy} = \tfrac{1}{12}m\ell^2 \quad I_{x'x'} = I_{y'y'} = \tfrac{1}{3}m\ell^2 \quad I_{zz} = 0$$

Moments of inertia for some common geometric solids



Parallel Axis Theorem

- The moment of inertia about any axis parallel to and at distance d away from the axis that passes through the centre of mass is:



$$I_O = I_G + md^2$$

- Where
  - I_G= moment of inertia for mass centre G
  - m = mass of the body
  - d = perpendicular distance between the parallel axes.

Radius of Gyration

Frequently tabulated data related to moments of inertia will be presented in terms of radius of gyration.

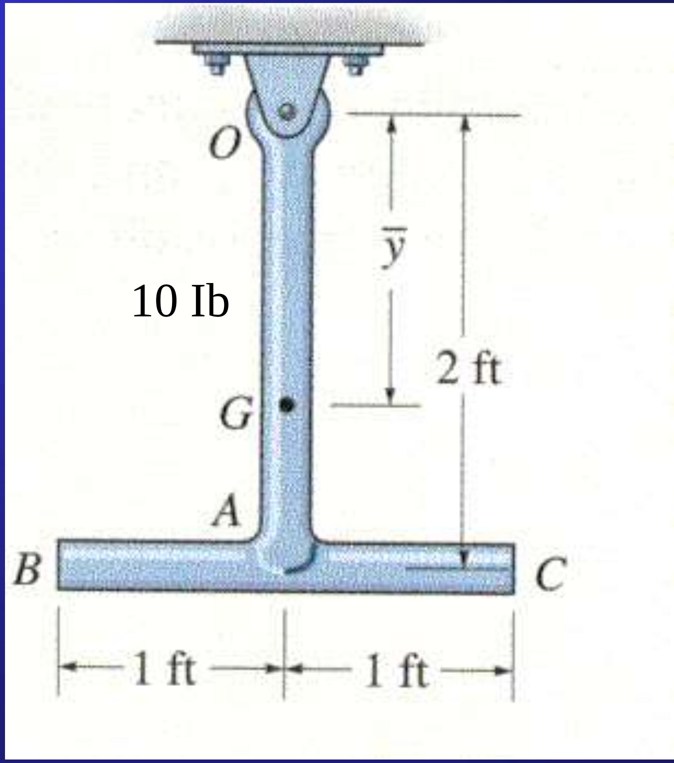
$$I = mk^2 \quad or \quad k = \sqrt{\frac{I}{m}}$$

Mass Center

$$\bar{y} = \frac{\sum \tilde{y} m}{\sum m}$$

Example

$$\bar{y} = \frac{\sum \tilde{y} m}{\sum m} = \frac{1(10/32.2) + 2(10/32.2)}{(10/32.2) + (10/32.2)} = 1.5\ \text{ft}$$



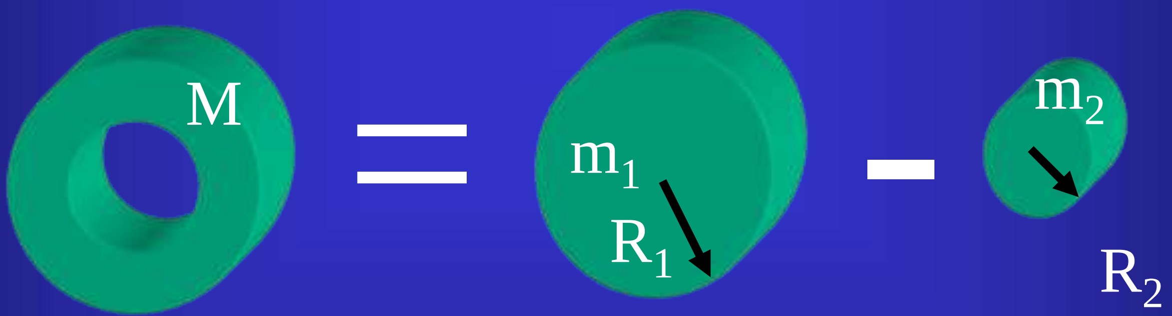
Moment of Inertia of Composite bodies

1. Divide the composite area into simple body.
2. Compute the moment of inertia of each simple body about its centroidal axis from table.
3. Transfer each centroidal moment of inertia to a parallel reference axis
4. The sum of the moments of inertia for each simple body about the parallel reference axis is the moment of inertia of the composite body.
5. Any cutout area has must be assigned a negative moment; all others are considered positive.

Moment of inertia of a hollow cylinder

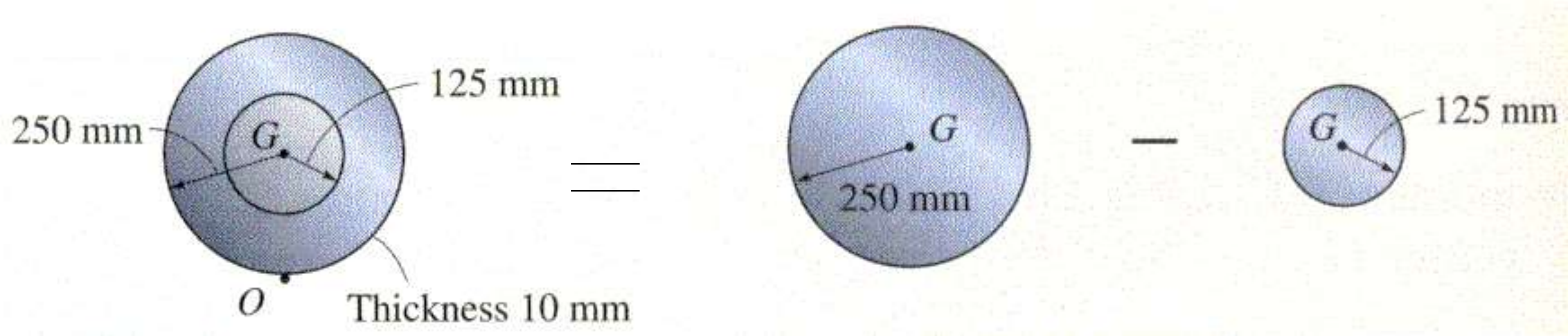
- Moment of Inertia of a solid cylinder
- A hollow cylinder

$$I = {}^{1}/_{2}\, mR^2$$



$$I = {}^{1}/_{2}\, m_1R_1^{\,2} - {}^{1}/_{2}\, m_2R_2^{\,2} = {}^{1}/_{2}\, M\,(R_1^{\,2} - R_2^{\,2})$$

Example 17-3



$$m_d = \rho_d V_d = 8000\frac{kg}{m^3}[\pi(0.25\,m)^2(0.01\,m)] = 15.71 \quad kg$$

$$I_G = \frac{1}{2}m\,r^2$$

$$(I_d)_O = \frac{1}{2}m_d r_d^{\,2} + m_d d^2$$

$$= \frac{1}{2}(15.71kg)(0.25m)^2 + (15.71kg)(0.25m)^2$$

$$= 1.473 \quad \text{kg.m}^2$$

$$m_h = \rho_h V_h = 8000\frac{kg}{m^3}[\pi(0.125\,m)^2(0.01\,m)] = 3.93 \quad kg$$

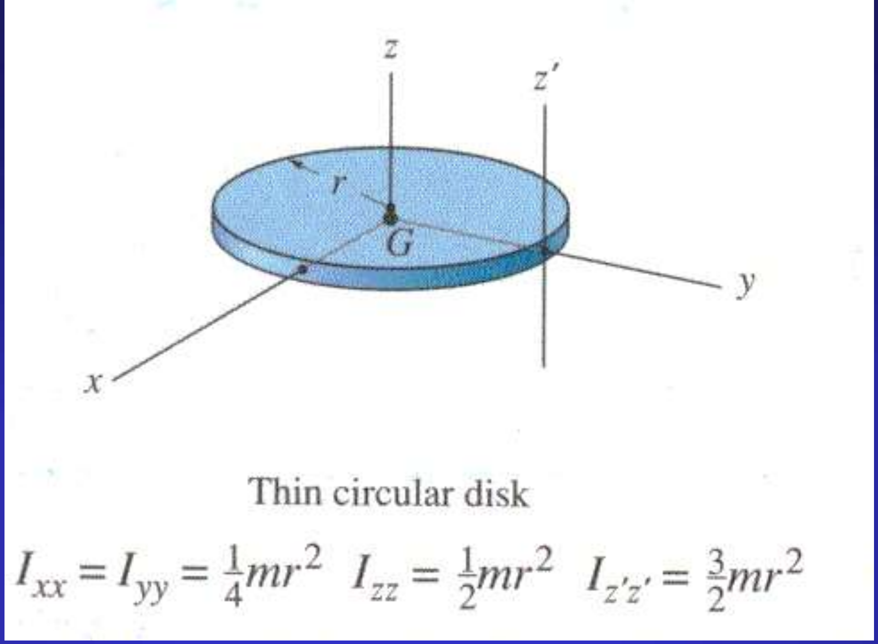
$$(I_h)_O = \frac{1}{2}m_h r_h^{\,2} + m_h d^2$$

$$= \frac{1}{2}(3.93kg)(0.125m)^2 + (3.93kg)(0.25m)^2$$

$$= 0.276 \quad \text{kg.m}^2$$

$$I_O = (I_d)_O - (I_h)_O$$

$$= 1.473 - 0.276 = 1.2\ kg.m^2$$



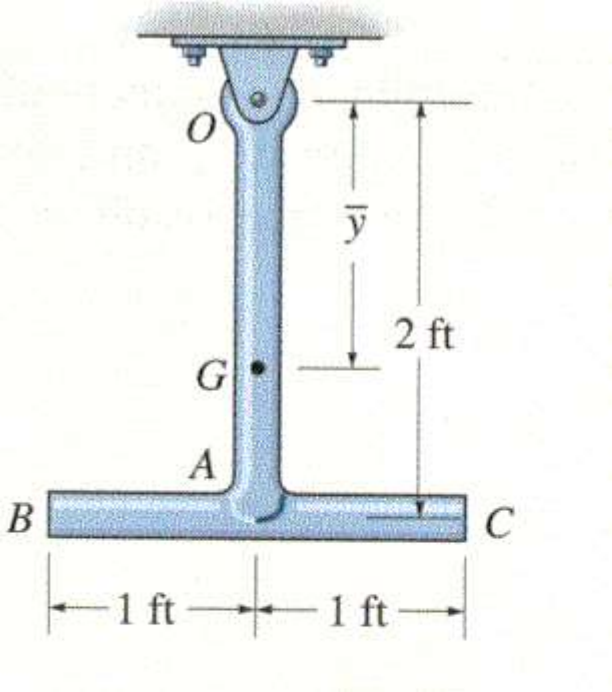
$$I_{z'z'} = \frac{3}{2} m_d r_d{}^2 - \left(\frac{1}{2} m_h r_h{}^2 + m_h d^2\right)$$

$$m_d = \rho_d V_d = 8000 \frac{kg}{m^3} [\pi (0.25\, m)^2 (0.01\, m)] = 15.71 \quad kg$$

$$m_h = \rho_h V_h = 8000 \frac{kg}{m^3} [\pi (0.125\, m)^2 (0.01\, m)] = 3.93 \quad kg$$

$$I_{z'z'} = \frac{3}{2}(15.71)(0.25)^2 - \left(\frac{1}{2}(3.93kg)(0.125m)^2 + (3.93kg)(0.25m)^2\right)$$

Example 17-4



$$(I_{OA})_O = \frac{1}{3}ml^2 = \frac{1}{3}(\frac{10\text{Ib}}{32.2\text{ft/s}})(2\text{ft})^2 = 0.414\ \text{slug.ft}^2$$

$$(I_{BC})_O = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(\frac{10}{32.2})(2)^2 + (\frac{10}{32.2})(2)^2$$
$$= 1.346\ \text{slug.ft}^2$$

$$I_O = 0.414 + 1.346 = 1.76\ \text{slug.ft}^2$$

$$\bar{y} = \frac{\sum \tilde{y}m}{\sum m}$$

$$\bar{y} = \frac{\sum \tilde{y}m}{\sum m} = \frac{1(10/32.2) + 2(10/32.2)}{(10/32.2) + (10/32.2)} = 1.5\ \text{ft}$$

$$I_O = I_G + md^2$$

$$1.76 = I_G + (\frac{20}{32.2})(1.5)^2$$

$$I_G = 0.362\ \text{slug.ft}^2$$

Thank You