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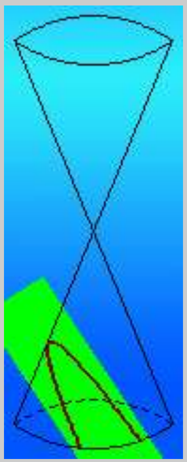
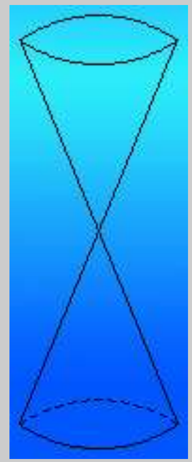
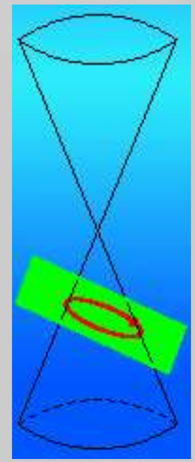
CONIC SECTIONS



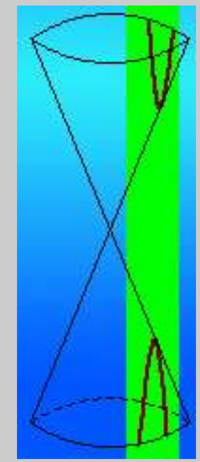
Parabola

Circle

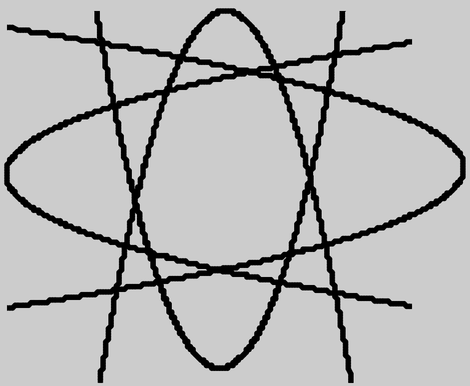


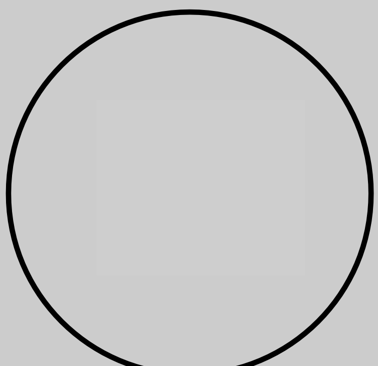


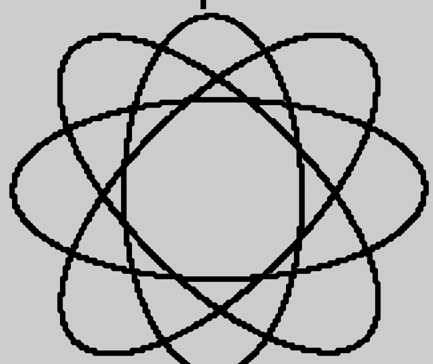
Ellipse

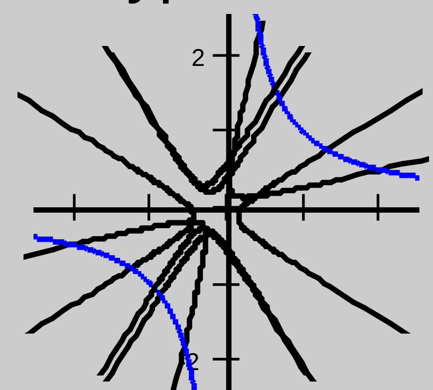


Hyperbola







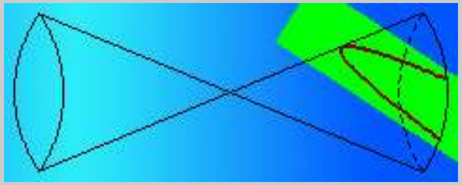


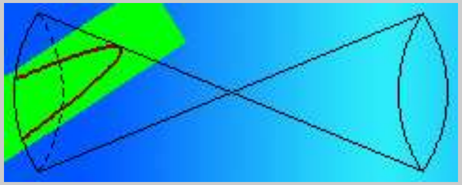
The quadratic relations that we studied in the beginning of the year were in the form of **$y = Ax^2 + Dx + F$**, where A, D, and F stand for constants, and $A \neq 0$.

This quadratic relation is a parabola, and it is the only one that can be a function.

It does *not* have to be a function, though.

A parabola is determined by a plane intersecting a cone and is therefore considered a conic section.



The general equation for all conic sections is:

$$\boxed{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$$

where A, B, C, D, E and F represent constants

and where the equal sign could be replaced by an

inequality sign.

When an equation has a "y^2" term and/or an "xy" term it is a quadratic relation instead of a quadratic function.

A Closer Look at Graphing Conics

Plot the graph of each relation. Select values of x and calculate the corresponding values of y until there are enough points to draw a smooth curve. Approximate all radicals to the nearest tenth.

1) $x^2 + y^2 = 25$

2) $x^2 + y^2 + 6x = 16$

3) $x^2 + y^2 - 4y = 21$

4) $x^2 + y^2 + 6x - 4y = 12$

5) Conclusions

$$x^2 + y^2 = 25$$

$$\boxed{x = 3}$$

$$3^2 + y^2 = 25$$

$$9 + y^2 = 25$$

$$y^2 = 16$$

$$\sqrt{y^2} = \sqrt{16}$$

$$|y| = 4$$

$$\boxed{y = \pm 4}$$

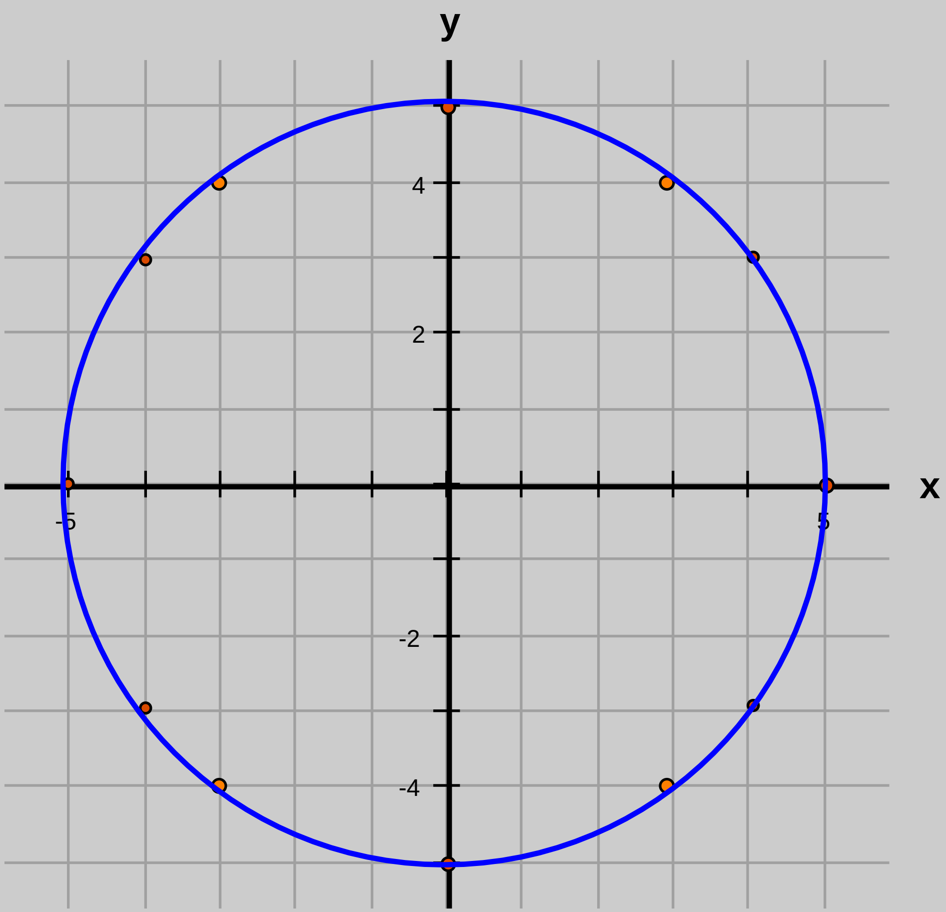
There are *2* points to graph: $\boxed{(3,4)}$ $\boxed{(3,-4)}$

Continue to solve in this manner, generating a table of values.

x	y
-5	0
-4	±3
-3	±4
-2	±4.6
-1	±4.9
0	±5
1	±4.9
2	±4.6
3	±4
4	±3
5	0

Graphing $x^2 + y^2 = 25$

x	y
-5	0
-4	±3
-3	±4
-2	±4.6
-1	±4.9
0	±5
1	±4.9
2	±4.6
3	±4
4	±3
5	0



$$x^2 + y^2 + 6x = 16$$

$$\boxed{x = 1}$$

$$1^2 + y^2 + 6(1) = 16$$

$$1 + y^2 + 6 = 16$$

$$y^2 = 9$$

$$\sqrt{y^2} = \sqrt{9}$$

$$|y| = 3$$

$$\boxed{y = \pm 3}$$

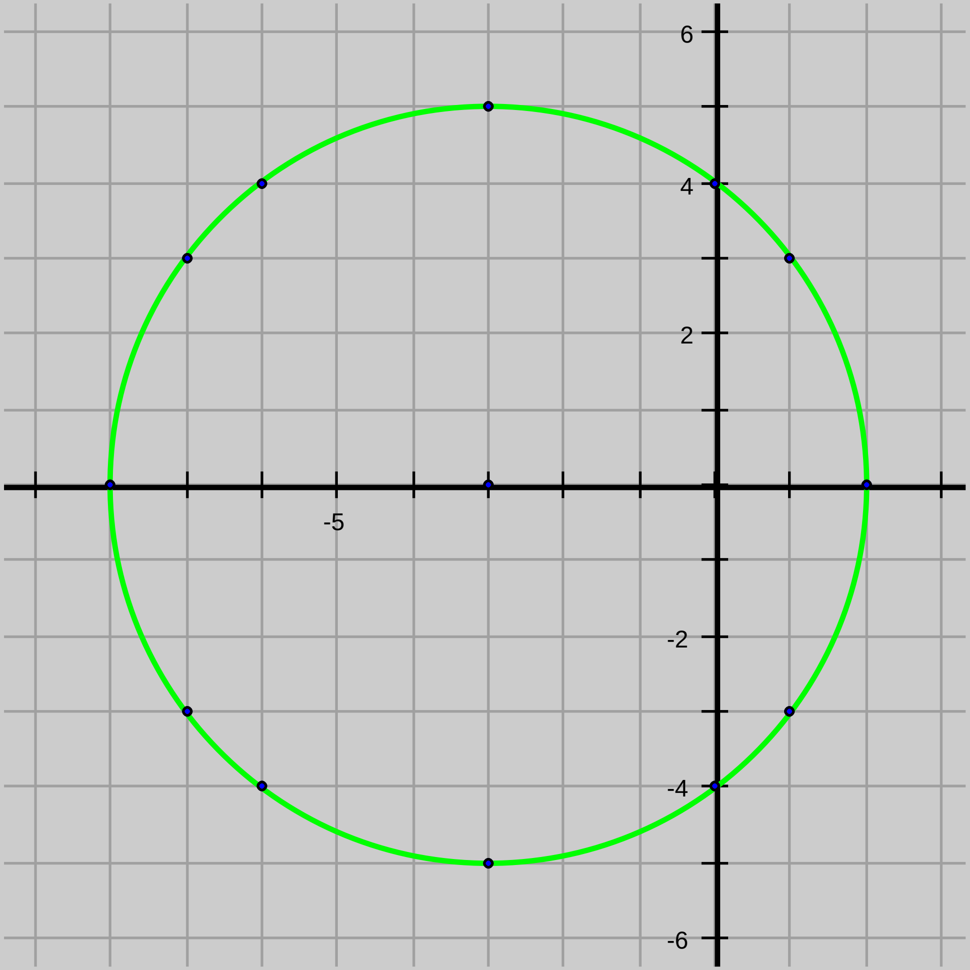
There are 2 points to graph: $\boxed{(1,3)}$ $\boxed{(1,-3)}$

Continue to solve in this manner, generating a table of values.

x	y
-8	0
-7	±3
-6	±4
-5	±4.6
-4	±4.9
-3	±5
-2	±4.9
-1	±4.6
0	±4
1	±3
2	0

Graphing $x^2 + y^2 + 6x = 16$

x	y
-8	0
-7	±3
-6	±4
-5	±4.6
-4	±4.9
-3	±5
-2	±4.9
-1	±4.6
0	±4
1	±3
2	0



$$x^2 + y^2 - 4y = 21$$

$\boxed{x = 3}$

$$3^2 + y^2 - 4y = 21$$

$$9 + y^2 - 4y = 21$$

$$y^2 - 4y - 12 = 0$$

$$(y - 6)(y + 2) = 0$$

$$y - 6 = 0 \text{ and } y + 2 = 0$$

$\boxed{y = 6 \text{ and } y = -2}$

There are 2 points to graph: $\boxed{(3,6)}$

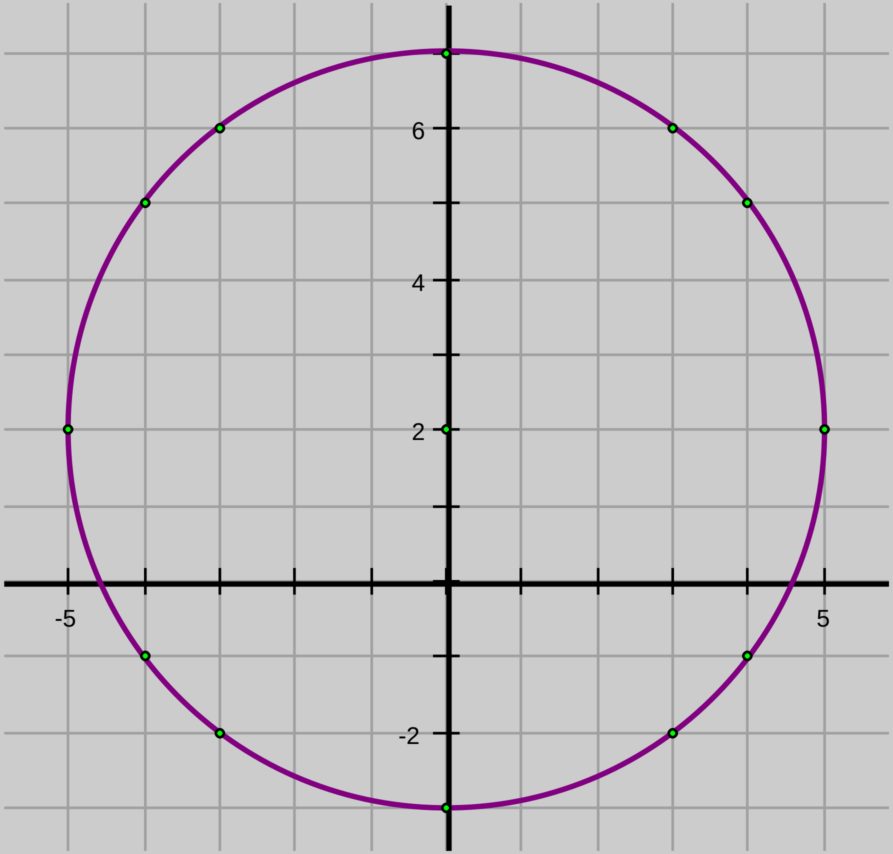
$\boxed{(3,-2)}$

Continue to solve in this manner, generating a table of values.

x	y
-5	2
-4	-1, 5
-3	-2, 6
-2	-2.6, 6.6
-1	-2.9, 6.9
0	-3, 7
1	-2.9, 6.9
2	-2.6, 6.6
3	-2, 6
4	-1, 5
5	2

Graphing $x^2 + y^2 - 4y = 21$

x	y
-5	2
-4	-1, 5
-3	-2, 6
-2	-2.6, 6.6
-1	-2.9, 6.9
0	-3, 7
1	-2.9, 6.9
2	-2.6, 6.6
3	-2, 6
4	-1, 5
5	2



$x^2 + y^2 + 6x - 4y = 12$

$\boxed{x = 1}$

$$1^2 + y^2 + 6(1) - 4y = 12$$

$$1 + y^2 + 6 - 4y = 12$$

$$y^2 - 4y - 5 = 0$$

$$(y - 5)(y + 1) = 0$$

$$y - 5 = 0 \text{ and } y + 1 = 0$$

$\boxed{y = 5 \text{ and } y = -1}$

There are *2* points to graph: $\boxed{(1,5)}$

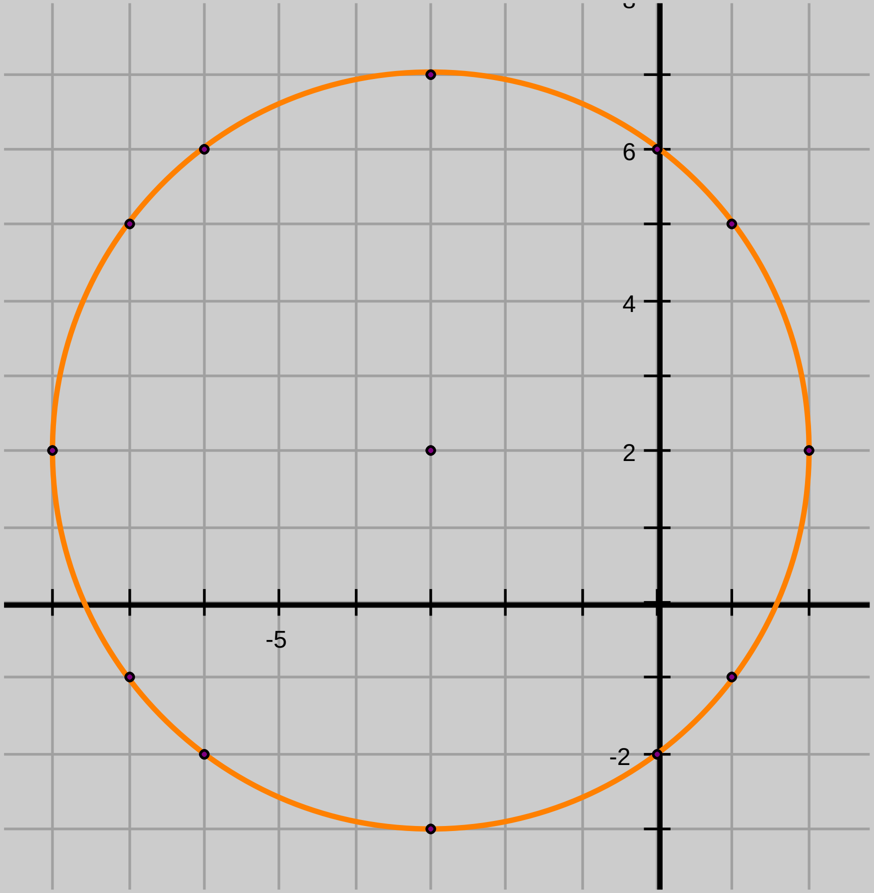
$\boxed{(1,-1)}$

Continue to solve in this manner, generating a table of values.

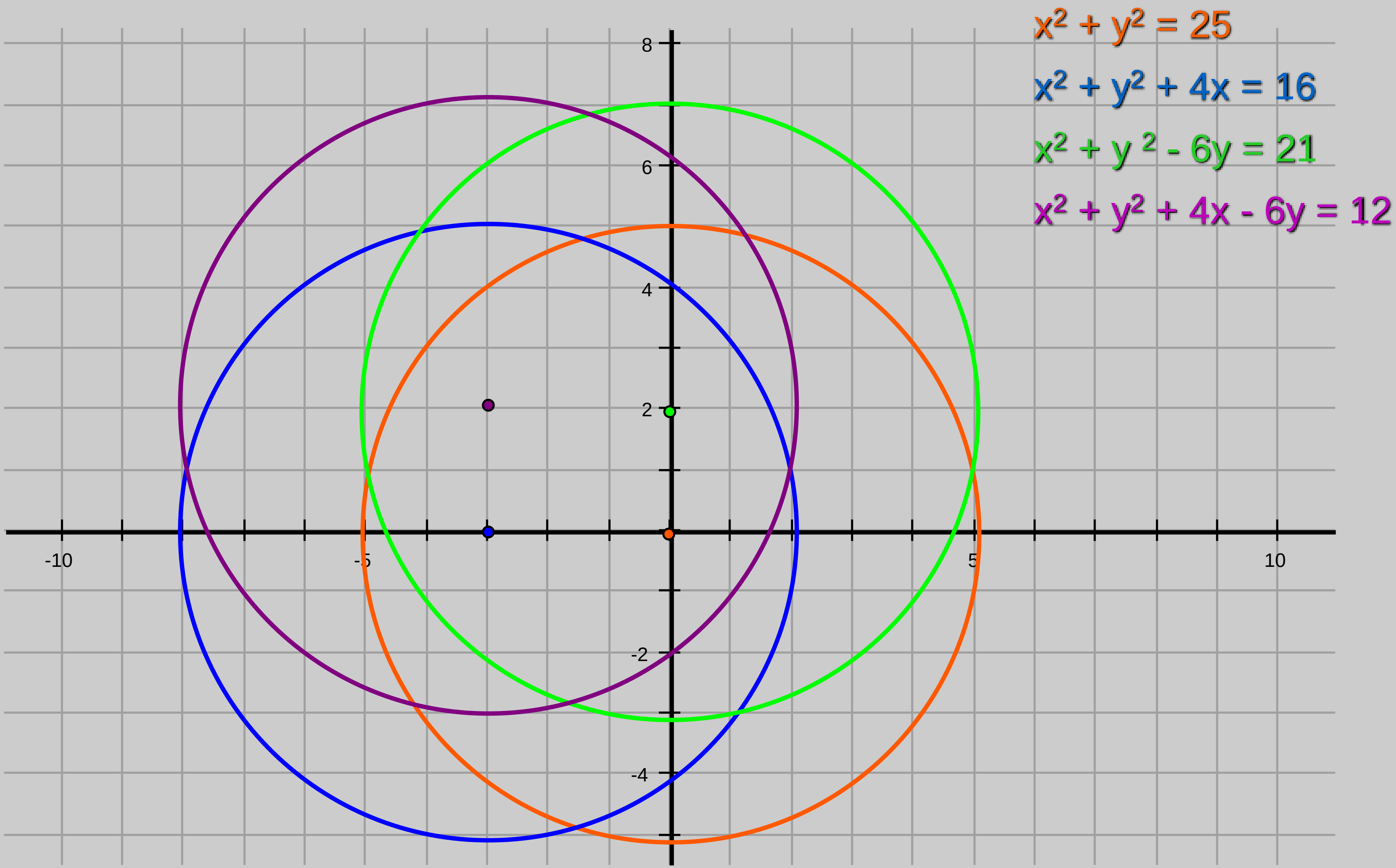
x	y
-8	2
-7	-1, 5
-6	-2, 6
-5	-2.6, 6.6
-4	-2.9, 6.9
-3	-3, 7
-2	-2.9, 6.9
-1	-2.6, 6.6
0	-2, 6
1	-1, 5
2	2

Graphing $x^2 + y^2 + 6x - 4y = 21$

x	y
-8	2
-7	-1, 5
-6	-2, 6
-5	-2.6, 6.6
-4	-2.9, 6.9
-3	-3, 7
-2	-2.9, 6.9
-1	-2.6, 6.6
0	-2, 6
1	-1, 5
2	2

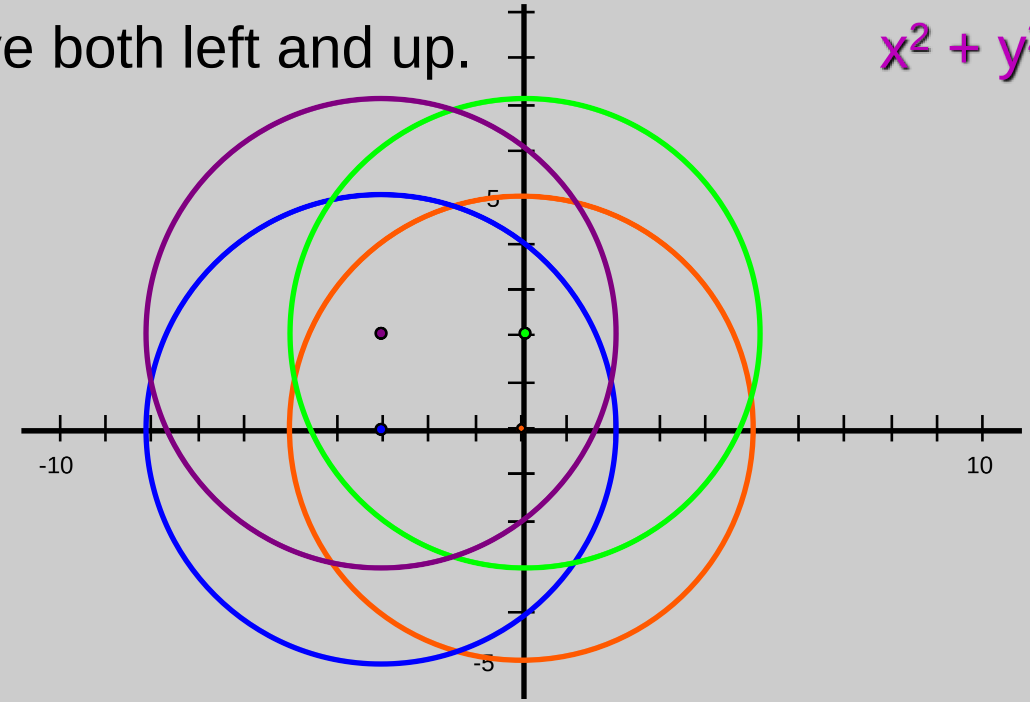


What conclusions can you draw about the shape and location of the graphs?



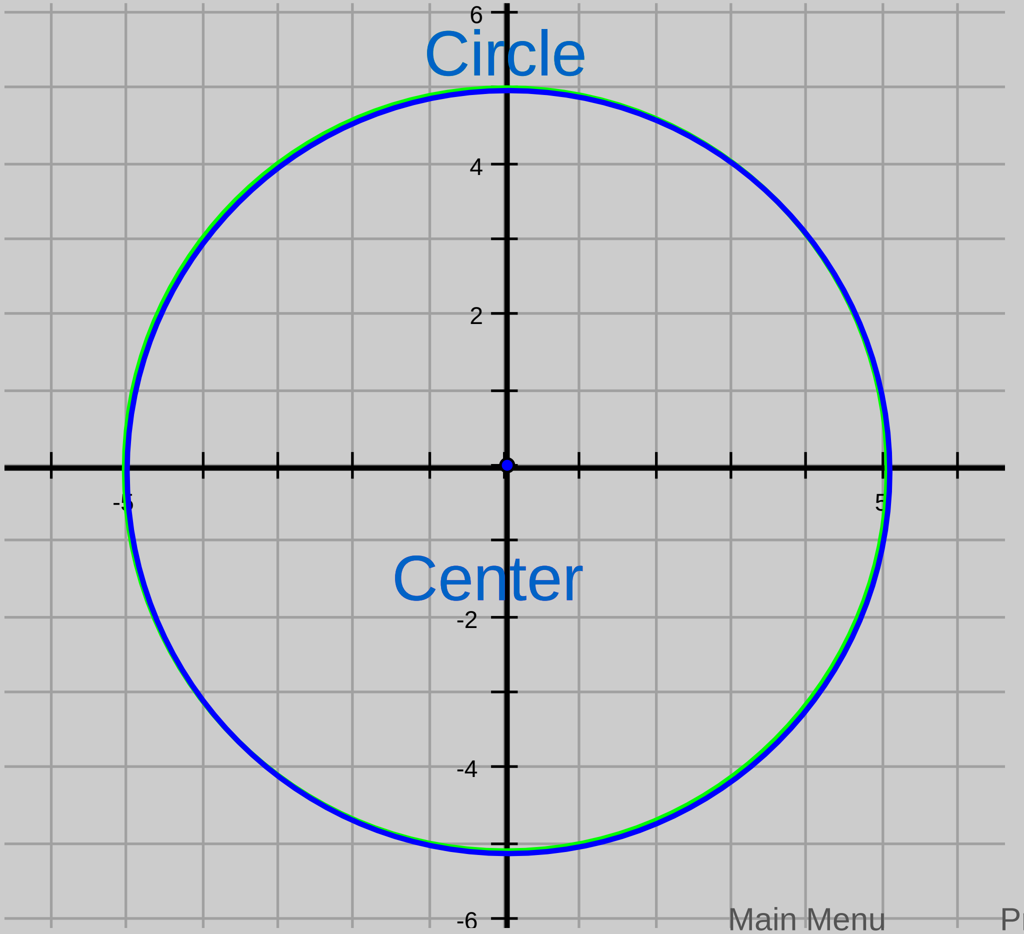
What conclusions can you draw about the shape and location of the graphs?

- All of the graphs are circles. $x^2 + y^2 = 25$
- As you add an x-term, the graph moves left. $x^2 + y^2 + 4x = 16$
- As you subtract a y-term, the graph moves up. $x^2 + y^2 - 6y = 21$
- You can move both left and up. $x^2 + y^2 + 4x - 6y = 12$

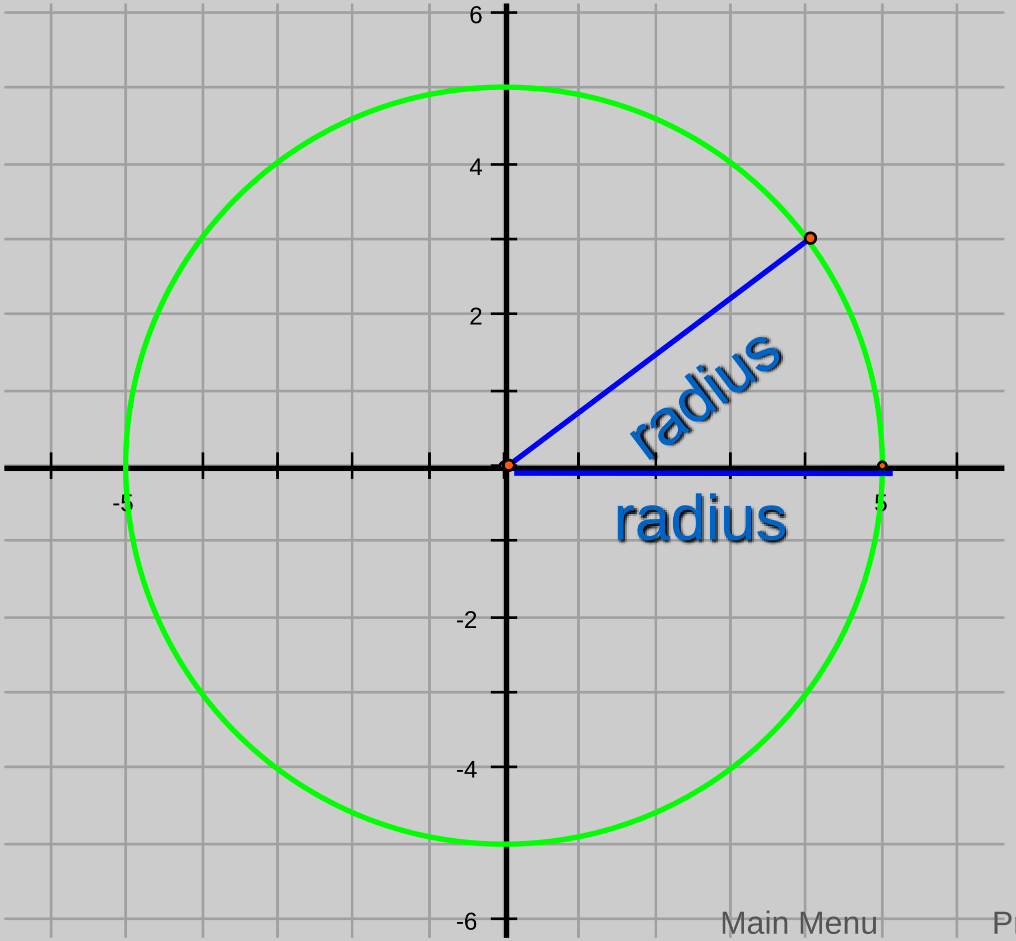


GEOMETRICAL DEFINITION OF A CIRCLE

A *circle* is the set of all points in a plane equidistant from a *fixed* point called the *center*.

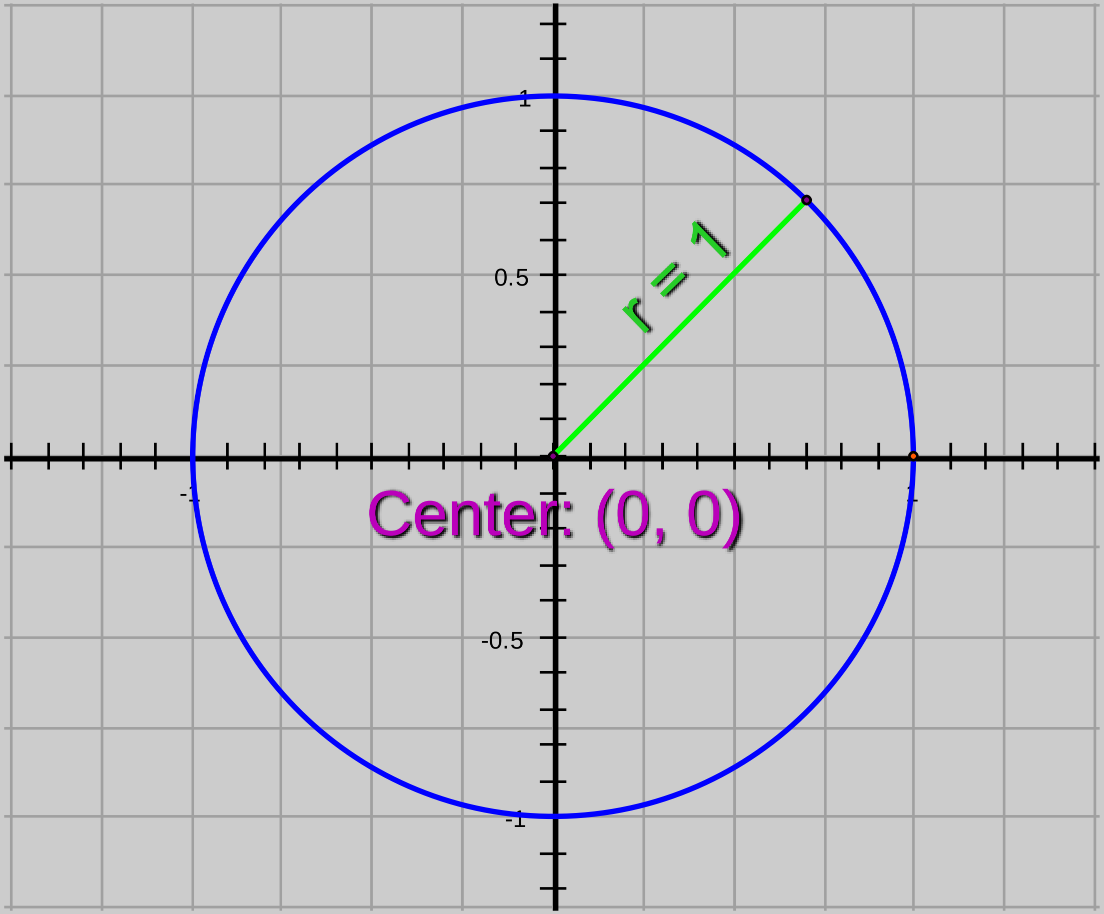


A *radius* is the segment whose endpoints are the center of the circle, and any point on the circle.

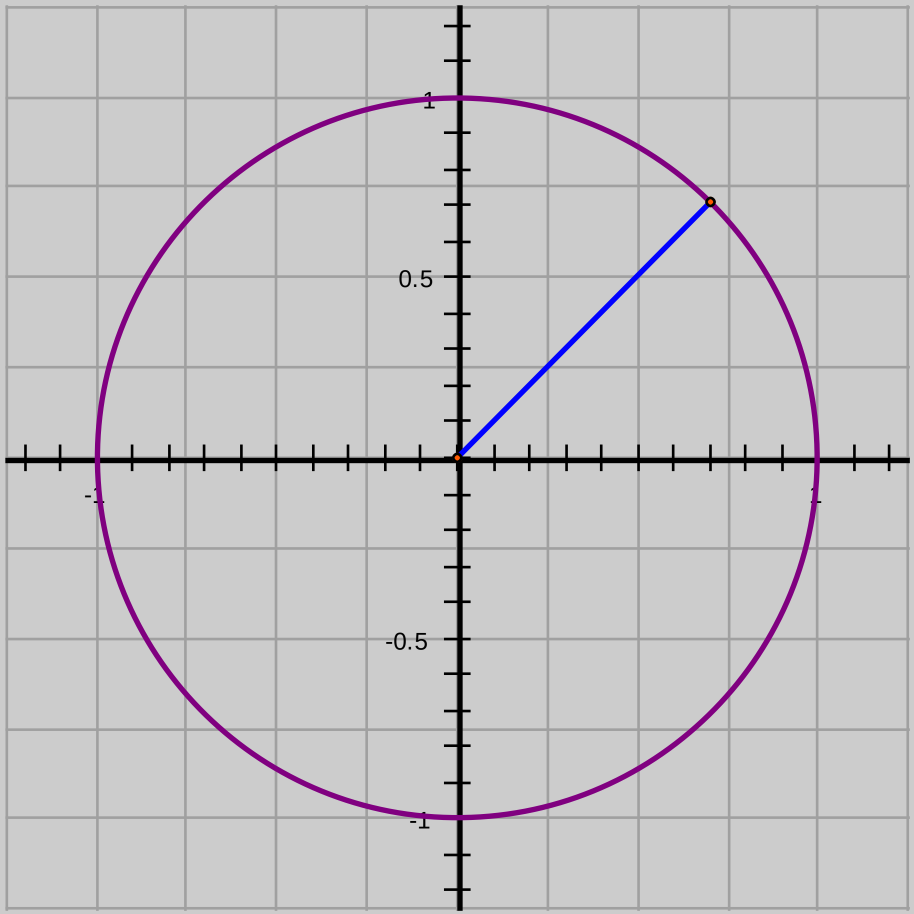


A *unit circle* is a circle with a *radius of 1* whose *center is at the origin*.

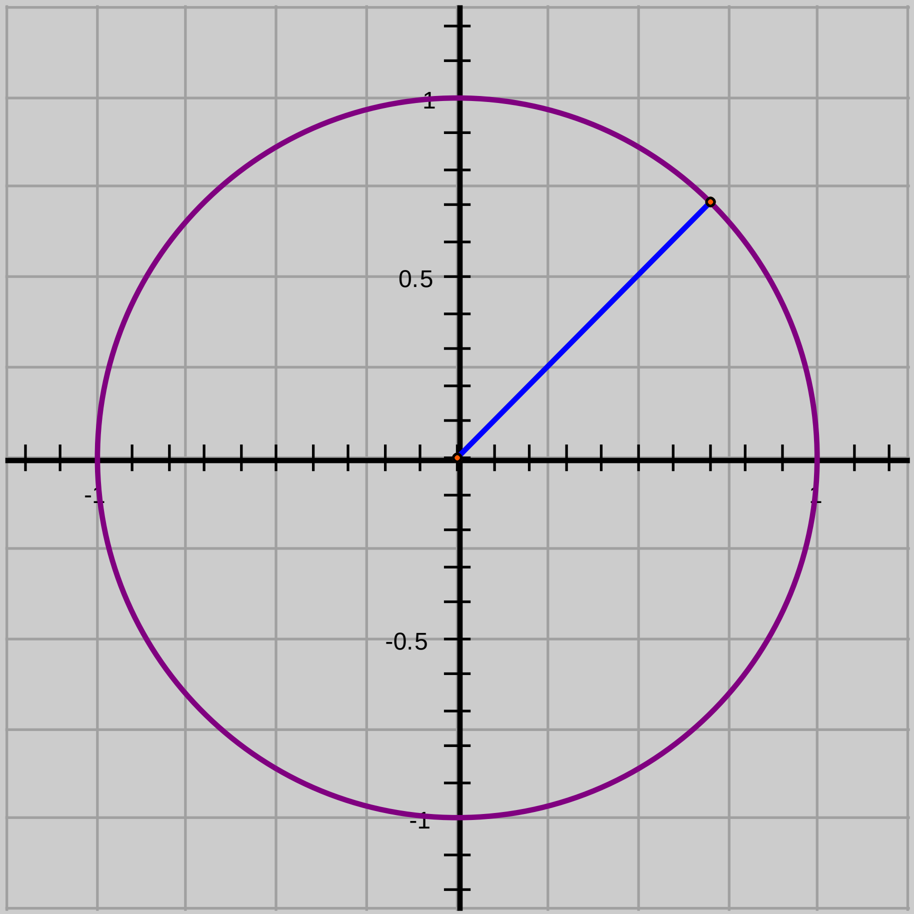
It is the circle on which all other circles are based.



The equation of a circle is derived from its ***radius***.

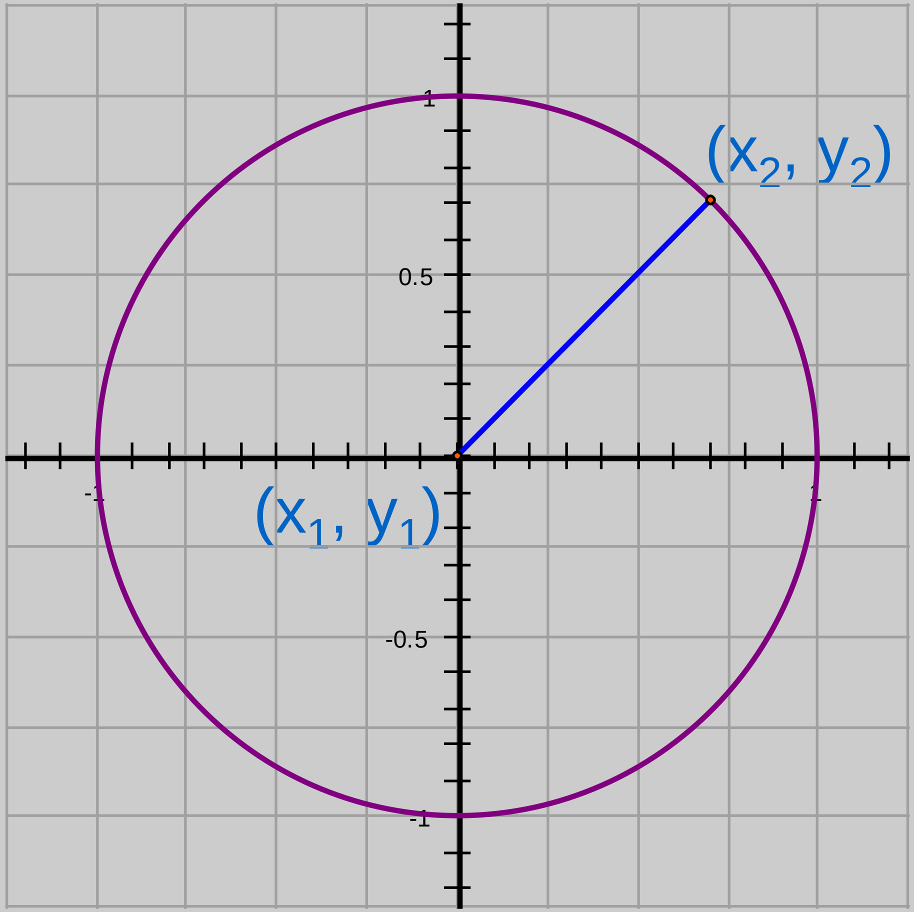


Use the ***distance formula*** to find an equation for x and y. This equation is also ***the equation for the circle***.



THE DISTANCE FORMULA

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let r for radius length replace D for distance.

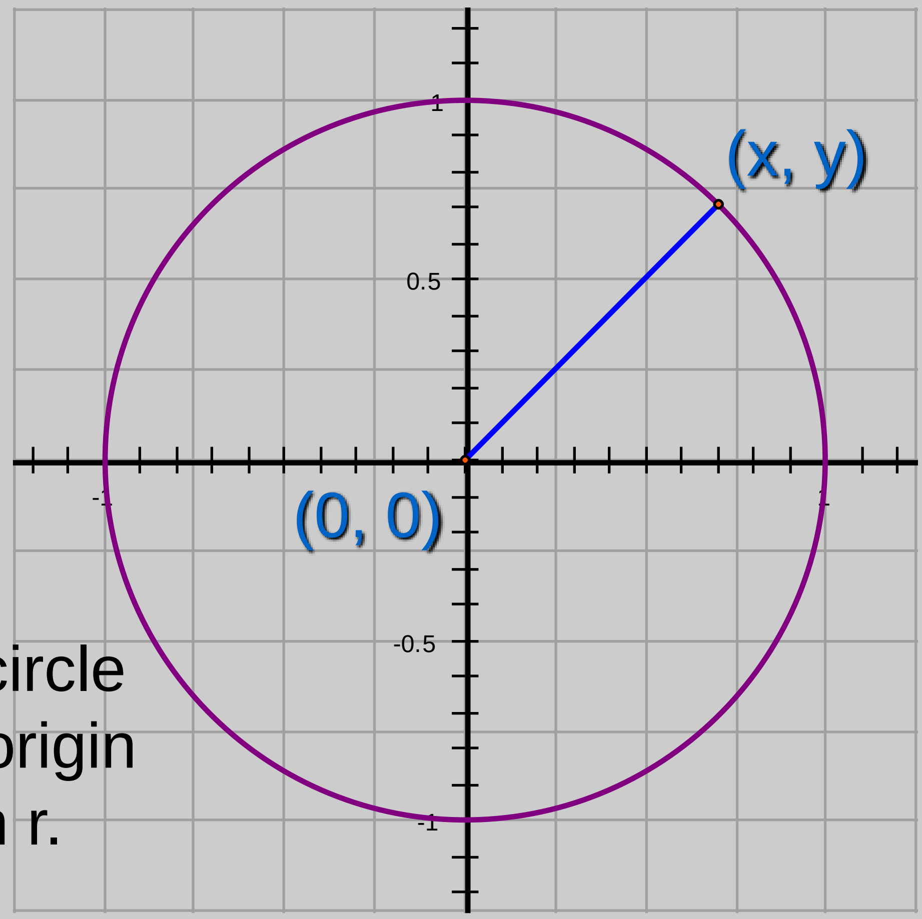
$$r = \sqrt{(x-0)^2 + (y-0)^2}$$

$$r = \sqrt{x^2 + y^2}$$

$$r^2 = \left(\sqrt{x^2 + y^2}\right)^2$$

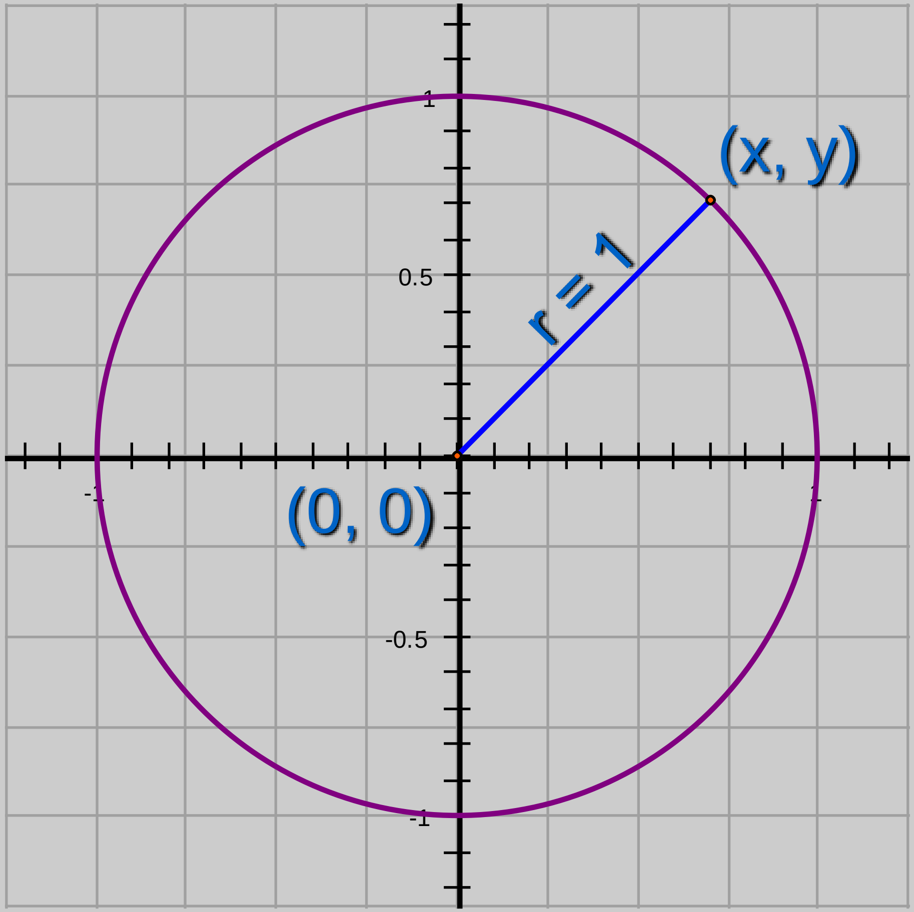
$$r^2 = x^2 + y^2$$

Is the equation for a circle with its center at the origin and a radius of length r.



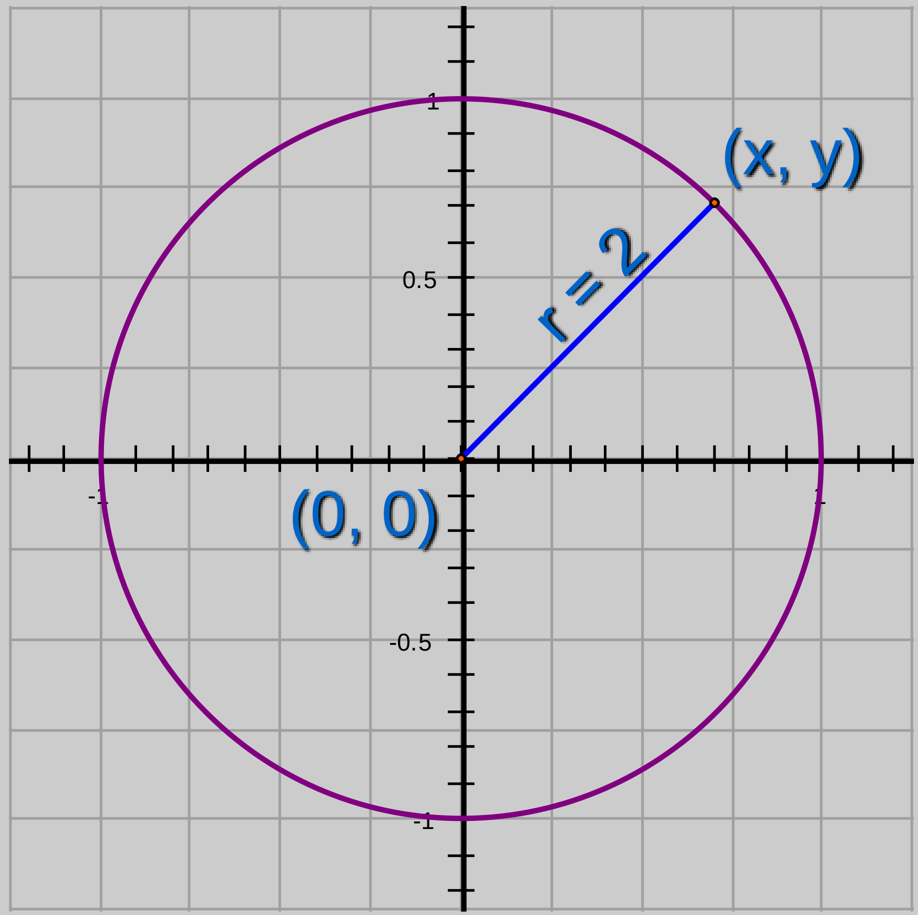
The unit circle therefore has the equation:

$$x^2 + y^2 = 1$$



If r = 2, then

$x^2 + y^2 = 4$



In order for a satellite to remain in a circular orbit above the Earth, the satellite must be 35,000 km above the Earth.

Write an equation for the orbit of the satellite. Use the center of the Earth as the origin and 6400 km for the radius of the earth.

$$(x - 0)^2 + (y - 0)^2 = (35000 + 6400)^2$$

$$x^2 + y^2 = 1,713,960,000$$

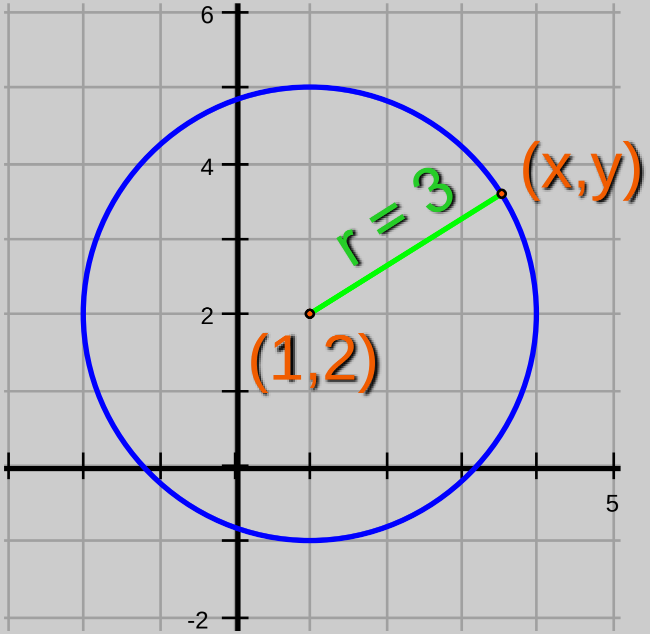
What will happen to the equation if the center is not at the origin?

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

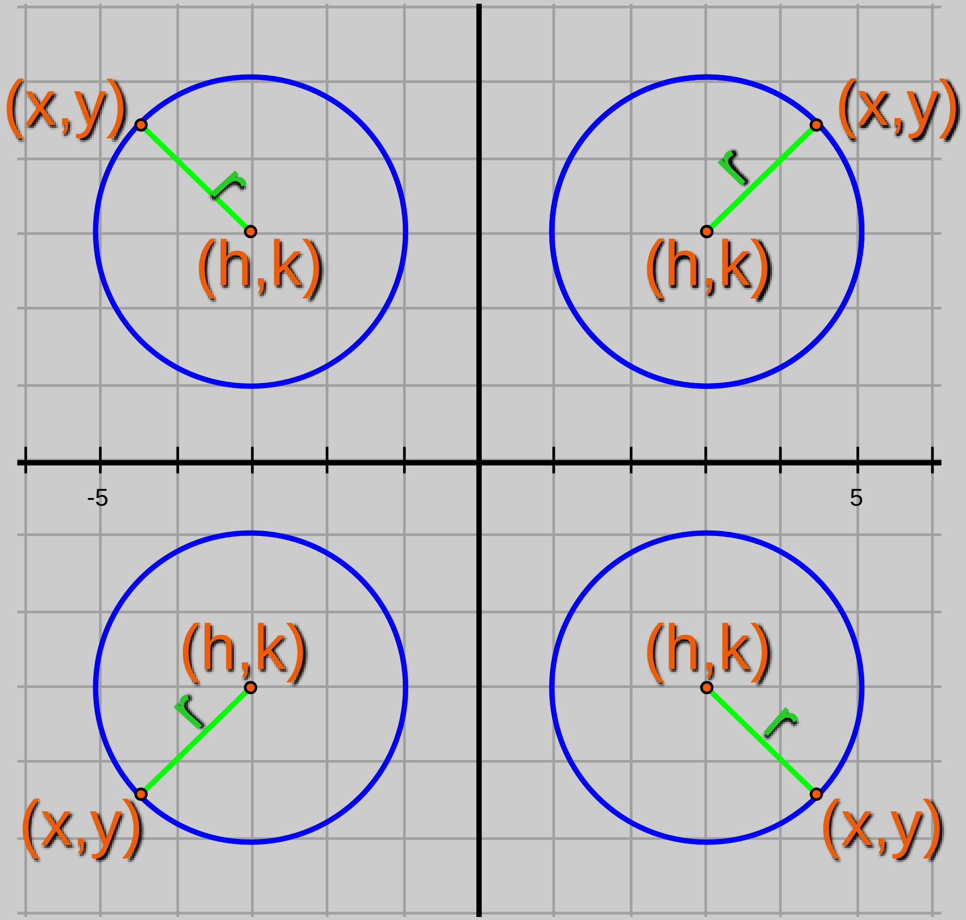
$$3 = \sqrt{(x-1)^2 + (y-2)^2}$$

$$3^2 = \left(\sqrt{(x-1)^2 + (y-2)^2}\right)^2$$

$$9 = (x-1)^2 + (y-2)^2$$



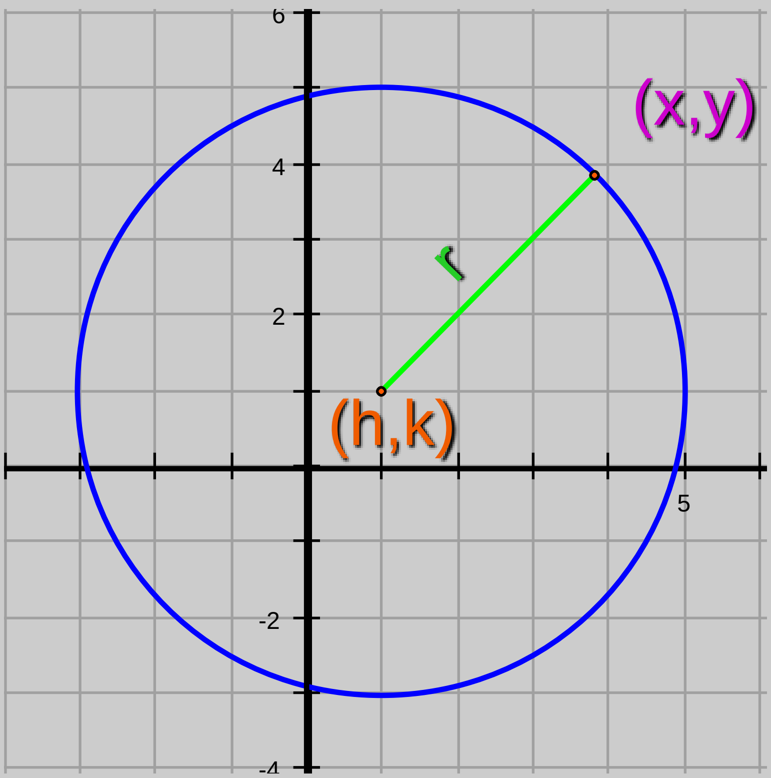
No matter where the circle is located, or where the center is, the equation is the same.



Assume (x, y) are the coordinates of a point on a circle.

The center of the circle is (h, k), and the radius is r.

Then the equation of a circle is: $(x - h)^2 + (y - k)^2 = r^2$.

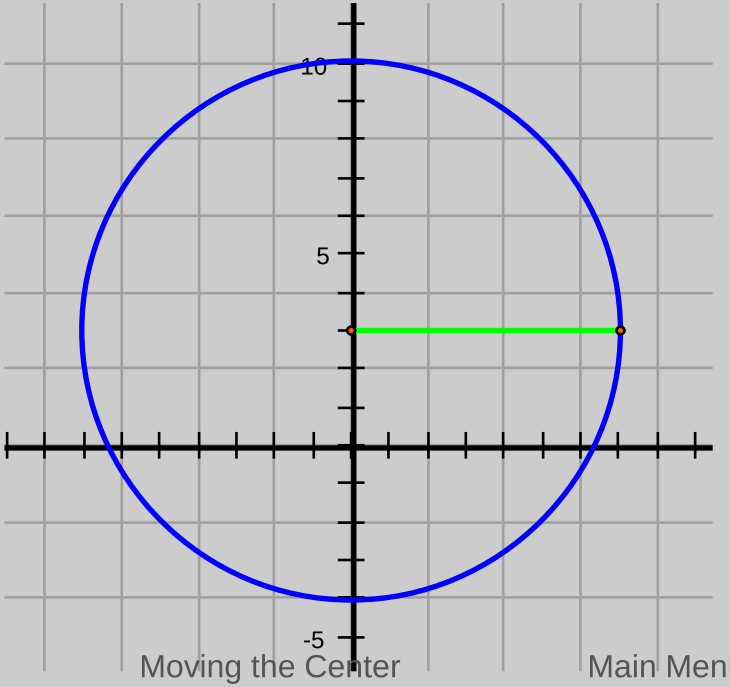


Write the equation of a circle with a center at (0, 3) and a radius of 7.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + (y - 3)^2 = 7^2$$

$$(x)^2 + (y - 3)^2 = 49$$



Find the equation whose diameter has endpoints of (-5, 2) and (3, 6).

First find the midpoint of the diameter using the midpoint formula. This will be the center.

MIDPOINT

$$M_{(x,y)} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M_{(x,y)} = \left(\frac{-5 + 3}{2}, \frac{2 + 6}{2} \right)$$

$$M_{(x,y)} = (-1, 4)$$

Find the equation whose diameter has endpoints of (-5, 2) and (3, 6).

Then find the length distance between the midpoint and one of the endpoints. This will be the radius.

DISTANCE FORMULA

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D = \sqrt{(-1 - 3)^2 + (4 - 6)^2}$$

$$D = \sqrt{(-4)^2 + (-2)^2}$$

$$D = \sqrt{16 + 4}$$

$$D = \sqrt{20}$$

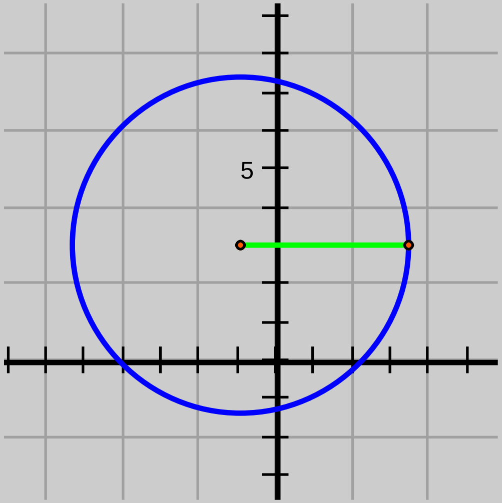
Find the equation whose diameter has endpoints of (-5, 2) and (3, 6).

Therefore the center is (-1, 4)

The radius is $\sqrt{20}$

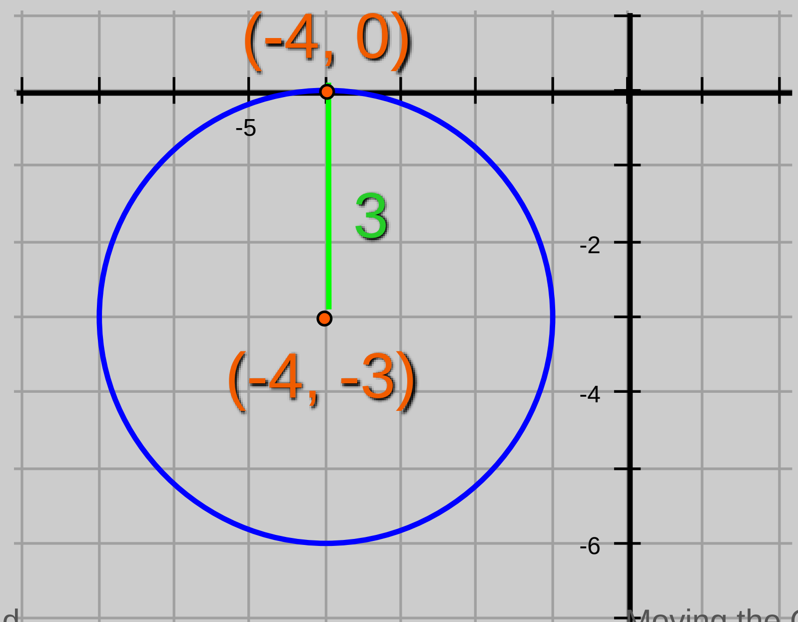
$$(x - -1)^2 + (y - 4)^2 = \sqrt{20}^2$$

$$(x + 1)^2 + (y - 4)^2 = 20$$



A line in the plane of a circle can *intersect* the circle in *1* or *2* points. A line that intersects the circle in *exactly one* point is said to be *tangent* to the circle. The line and the circle are considered tangent to each other at this point of intersection.

Write an equation for a circle with center (-4, -3) that is tangent to the x-axis. A *diagram* will help.



A radius is always *perpendicular* to the tangent line.

$(x + 4)^2 + (y + 3)^2 = 9$

The standard form equation for all conic sections is:

$$\boxed{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$$

where A, B, C, D, E and F represent constants and where the equal sign could be replaced by an inequality sign.

How do you put a standard form equation into graphing form?

The transformation is accomplished through *completing the square*.

Graph the relation $x^2 + y^2 - 10x + 4y + 13 = 0$.

1. Move the F term to the other side.

$$x^2 + y^2 - 10x + 4y = -13$$

2. Group the x-terms and y-terms together

$$x^2 - 10x + y^2 + 4y = -13$$

3. Complete the square for the x-terms and y-terms.

$$x^2 - 10x + y^2 + 4y = -13$$

$$\boxed{\frac{-10}{2} = -5} \quad \boxed{\frac{4}{2} = 2}$$

$$\boxed{(-5)^2 = 25} \quad \boxed{(2)^2 = 4}$$

$$x^2 - 10x + 25 + y^2 + 4y + 4 = -13 + 25 + 4$$

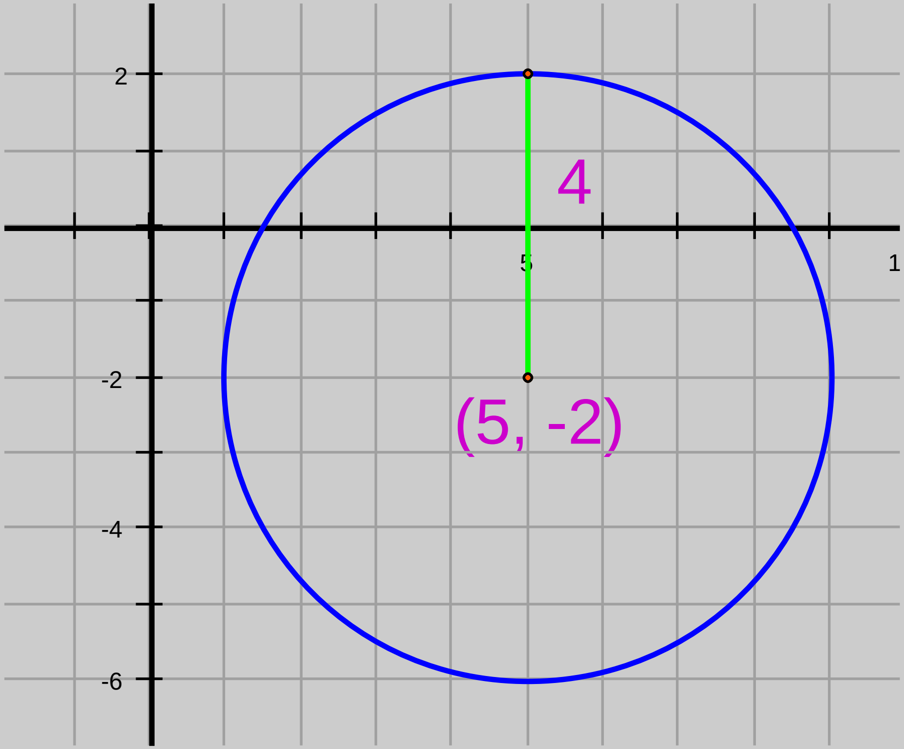
$$(x - 5)^2 + (y + 2)^2 = 16$$

$$(x - 5)^2 + (y + 2)^2 = 16$$

$$(x - 5)^2 + (y + 2)^2 = 4^2$$

center: (5, -2)

radius = 4



What if the relation is an inequality?

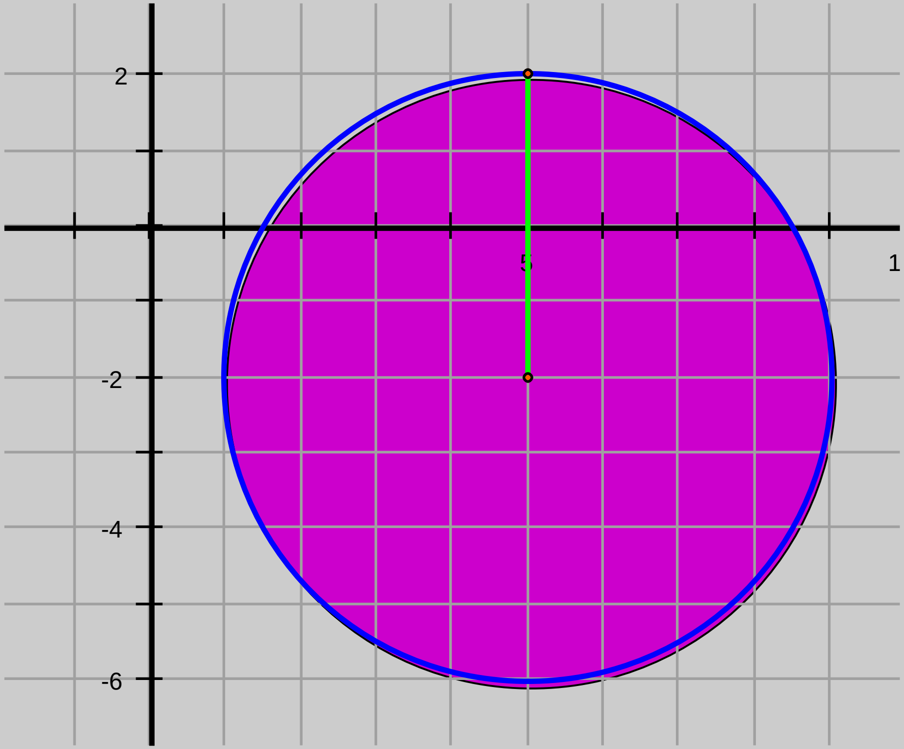
$$x^2 + y^2 - 10x + 4y + 13 < 0$$

Do the same steps to transform it to graphing form.

$$(x - 5)^2 + (y + 2)^2 < 4^2$$

This means the values are inside the circle.

The values are less than the radius.

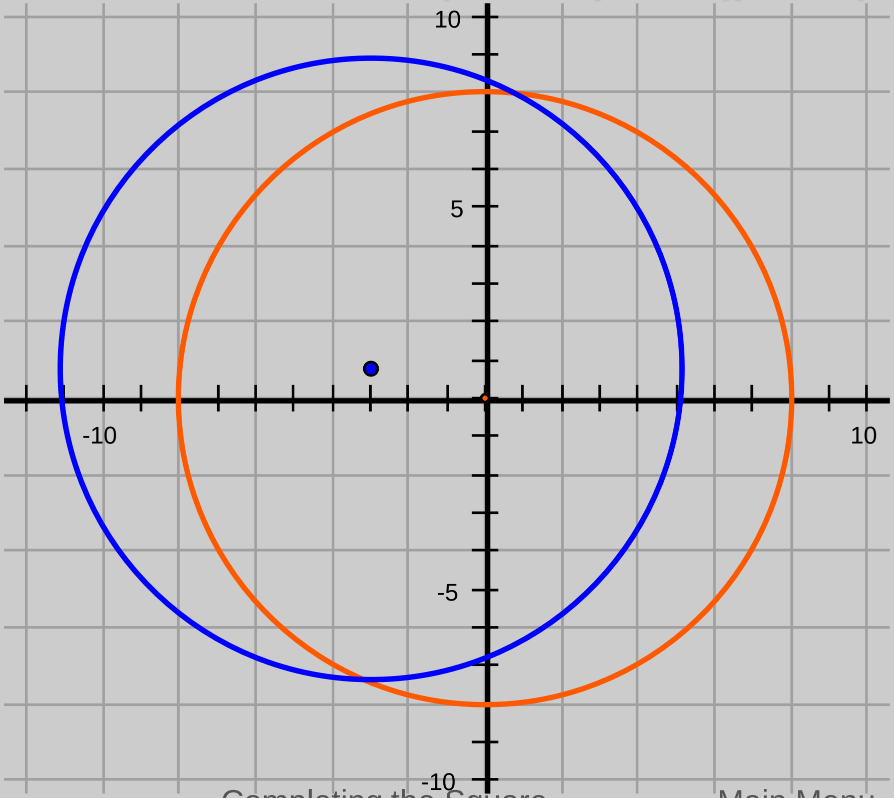


Write $x^2 + y^2 + 6x - 2y - 54 = 0$ in graphing form. Then describe the transformation that can be applied to the graph of $x^2 + y^2 = 64$ to obtain the graph of the given equation.

1. $x^2 + y^2 + 6x - 2y = 54$
2. $x^2 + 6x + y^2 - 2y = 54$
3. $(^6/_2) = 3$ $(^{-2}/_2) = -1$
4. $(3)^2 = 9$ $(-1)^2 = 1$
5. $x^2 + 6x + 9 + y^2 - 2y + 1 = 54 + 9 + 1$
6. $(x + 3)^2 + (y - 1)^2 = 64$
7. $(x + 3)^2 + (y - 1)^2 = 8^2$
8. center: (-3, 1) radius = 8

Write $x^2 + y^2 + 6x - 2y - 54 = 0$ in graphing form. Then describe the transformation that can be applied to the graph of $x^2 + y^2 = 64$ to obtain the graph of the given equation.

$x^2 + y^2 = 64$ is translated 3 units left and one unit up to become $(x + 3)^2 + (y - 1)^2 = 64$.



The graph of a quadratic relation *will* be a ***circle*** if the ***coefficients*** of the x^2 term and y^2 term are ***equal*** (and the xy term is zero).