

# *Conics*

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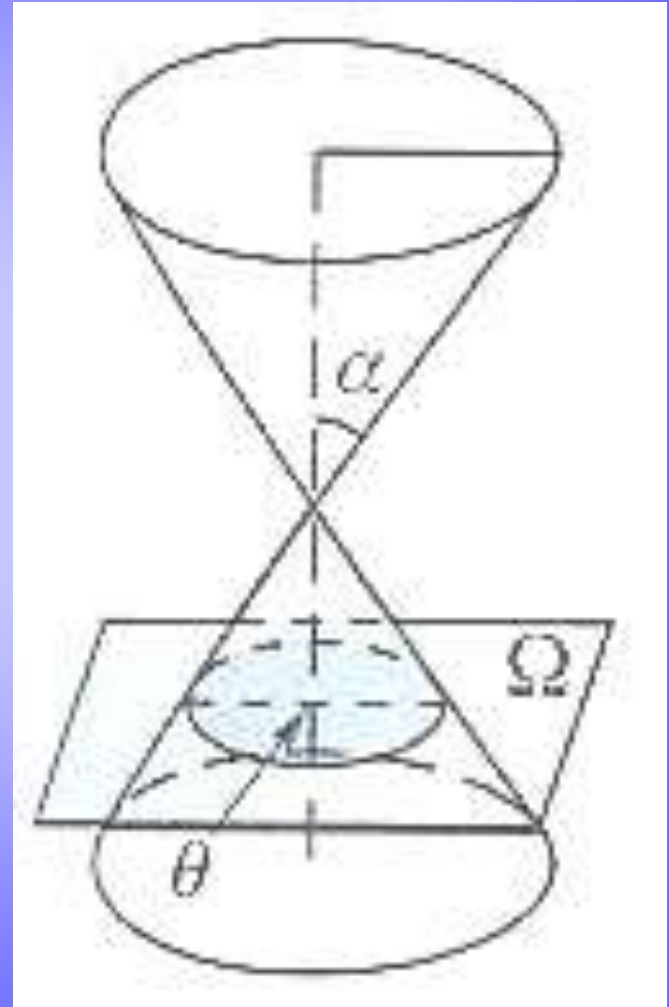
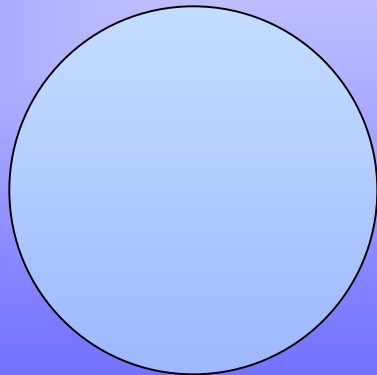
# Conic Sections

## (1) Circle

A circle is formed when

$$\theta = \frac{\pi}{2}$$

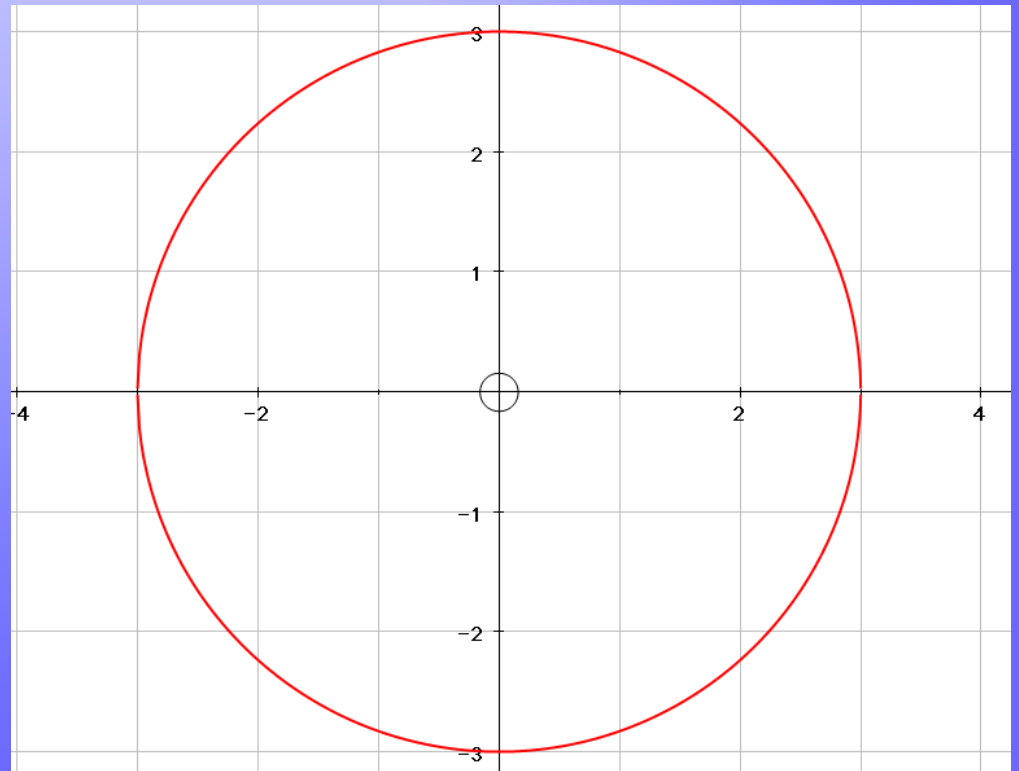
i.e. when the plane  $\Omega$  is perpendicular to the axis of the cones.



# Equation of a circle

- We should recall that the equation of a circle is given by:

$$x^2 + y^2 = r^2$$



What is the equation of this circle?

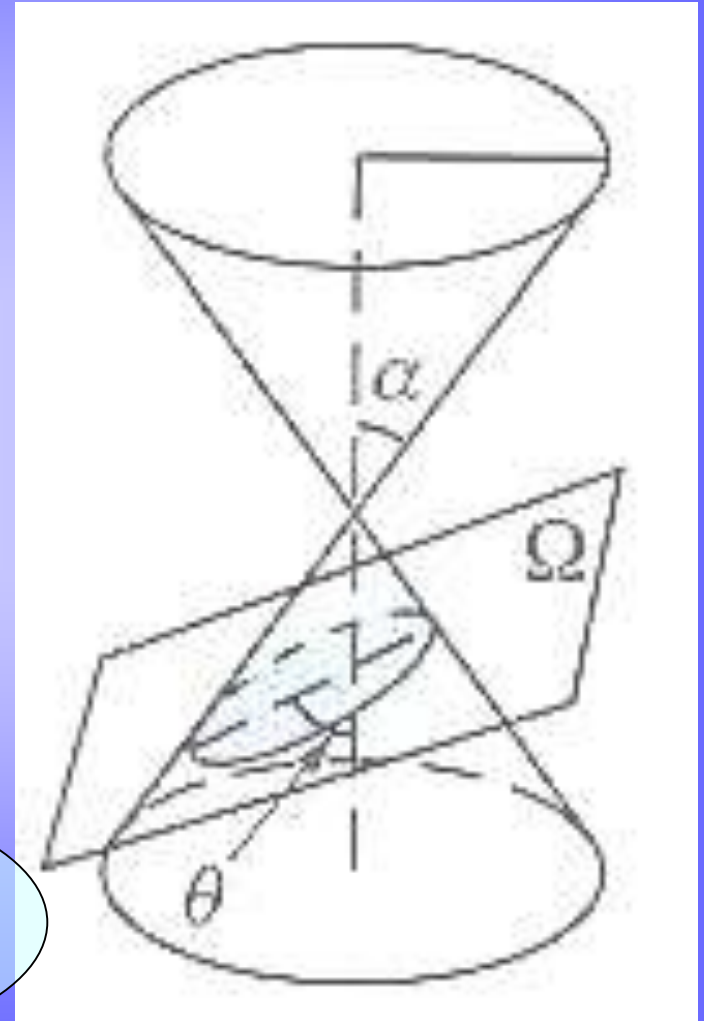
# Conic Sections

## (2) Ellipse

An ellipse is formed when

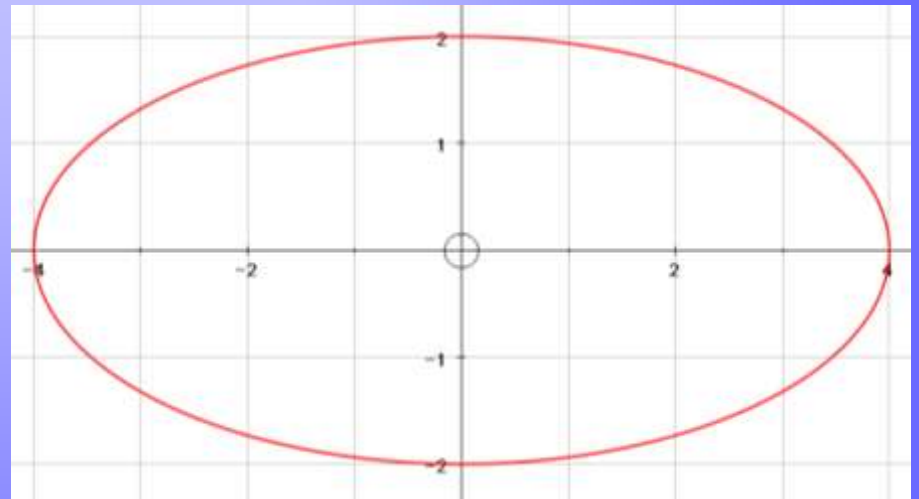
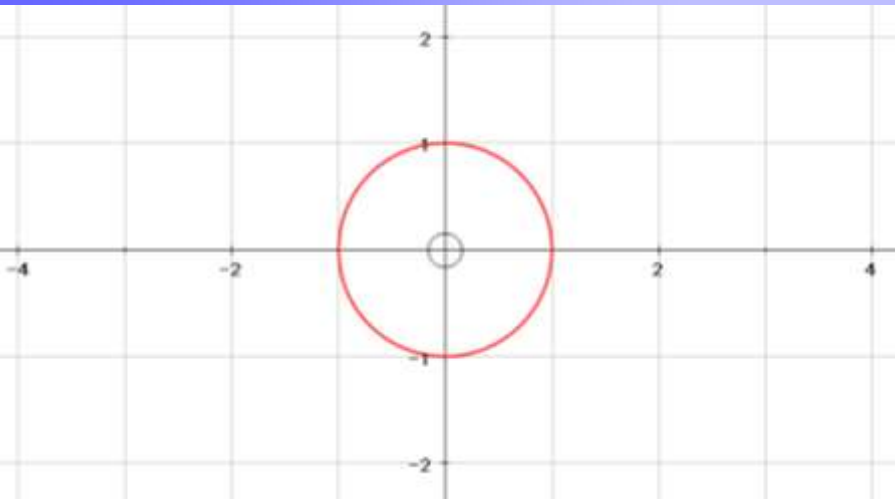
$$\alpha < \theta < \frac{\pi}{2}$$

i.e. when the plane  $\Omega$  cuts only one of the cones, but is neither perpendicular to the axis nor parallel to the a generator.



# Equation of an ellipse

- What transformation would change the circle into the ellipse?



What is the equation of the ellipse?  $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

Can you write it another way?  $\frac{x^2}{16} + \frac{y^2}{4} = 1$   $x^2 + 4y^2 = 16$

# Equation of an ellipse

Think of an ellipse as a unit circle stretched by factor  $a$  in the x-direction and factor  $b$  in the y-direction

This means the equation can be given by...

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \quad \text{or} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where would it cut the x and y axis?

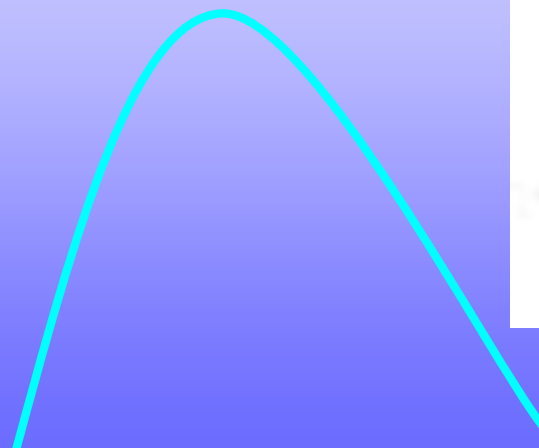
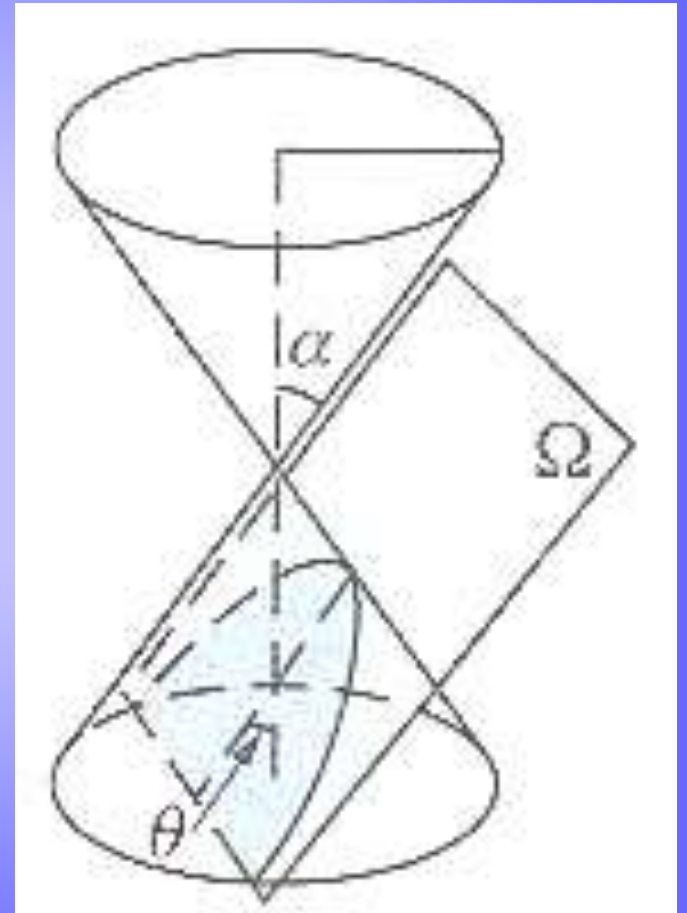
## Conic Sections

### (3) Parabola

A parabola is formed when

$$\theta = \alpha$$

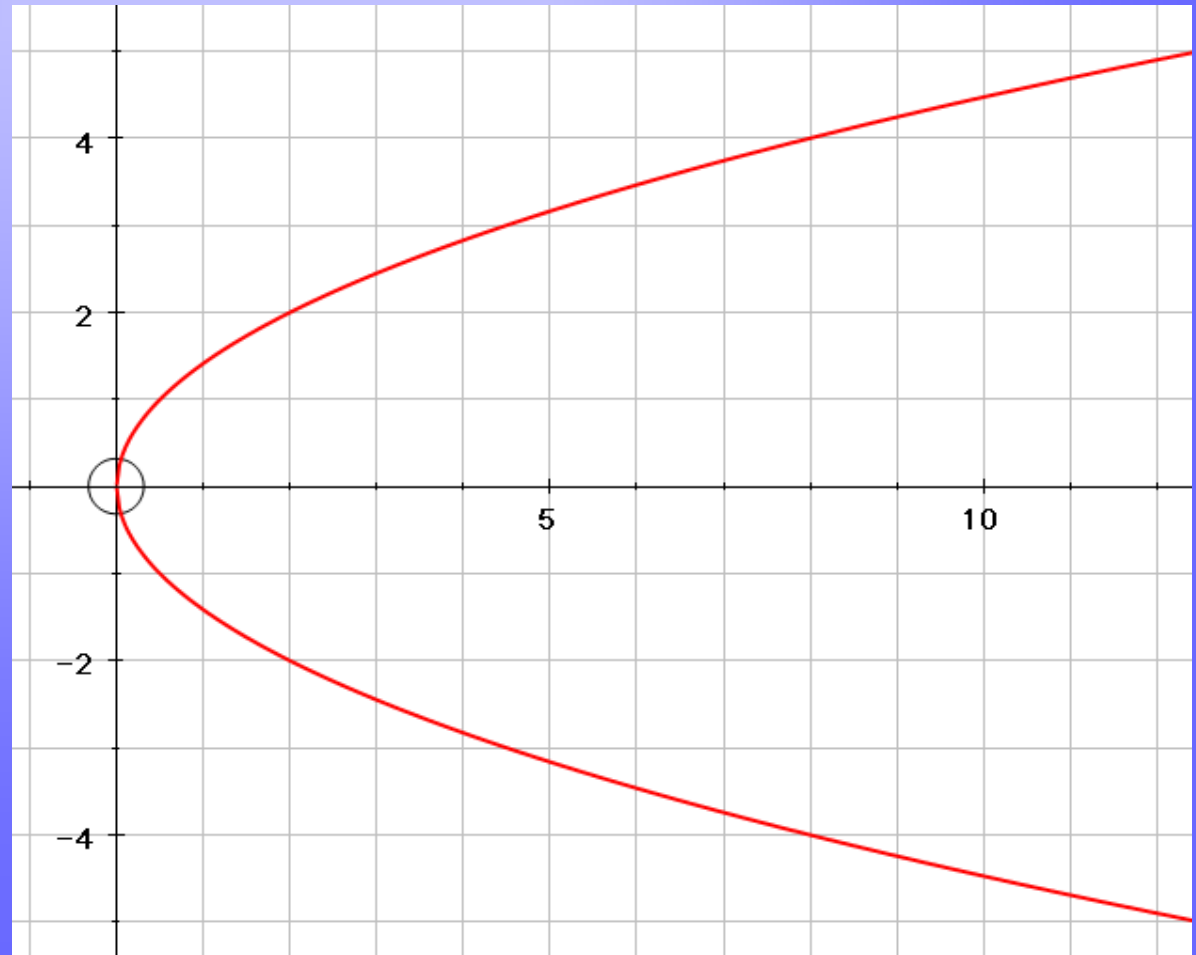
i.e. when the plane  $\Omega$  is parallel to a generator.



# Parabola

The general equation of a parabola is given by:

$$y^2 = 4ax$$





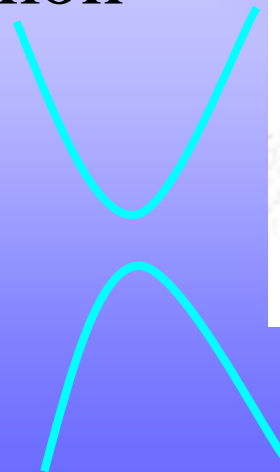
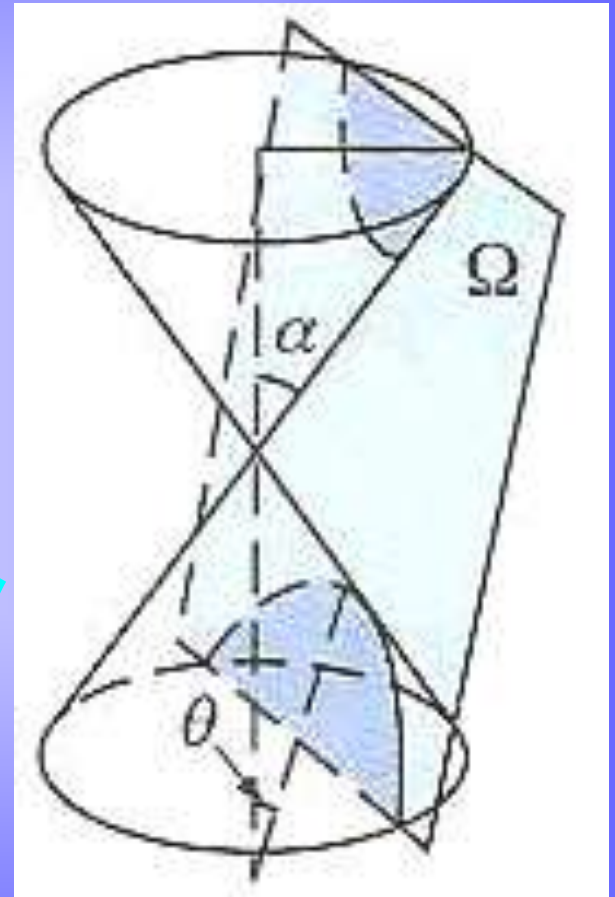
# Conic Sections

## (4) Hyperbola

A hyperbola is formed when

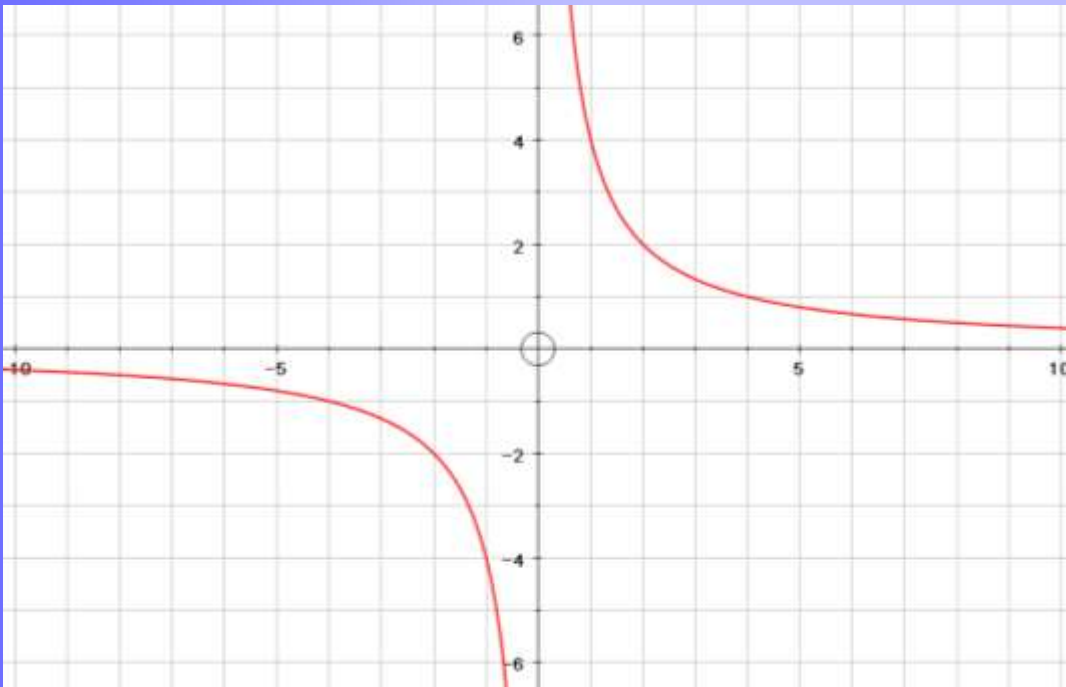
$$0 \leq \theta < \alpha$$

i.e. when the plane  $\Omega$  cuts both the cones, but does not pass through the common vertex.



# Hyperbola (rectangular)

What type of graph is the one below? Can you write down a general equation for this type of graph?



It is a reciprocal graph

$$y = \frac{c^2}{x}$$

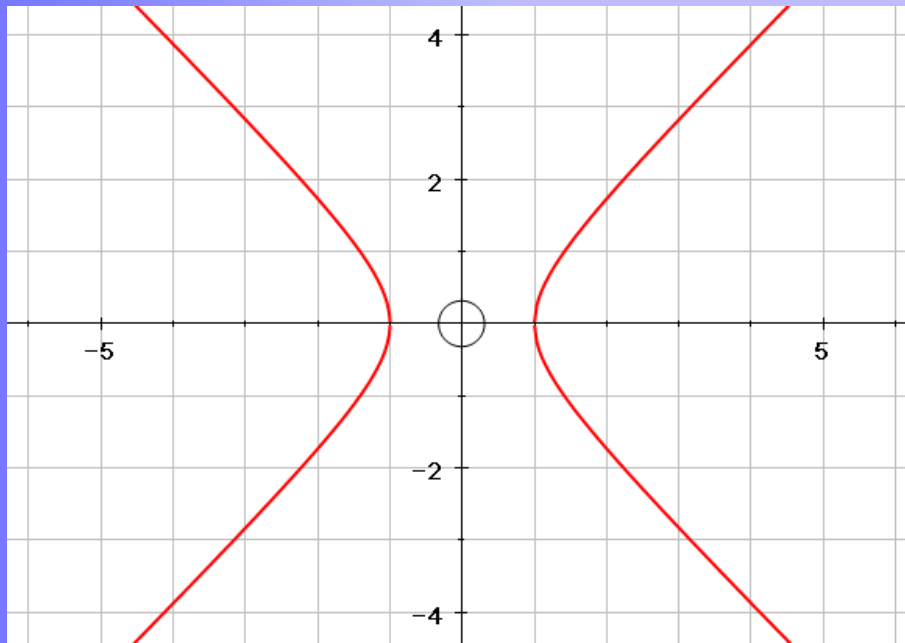
$$xy = c^2$$

The point closest to the origin has co-ordinates (c,c).

What are the asymptotes of this graph?

# Hyperbola (rectangular)

It can be shown using a matrix transformation that after a rotation of  $45^\circ$ , our hyperbola has the general equation..



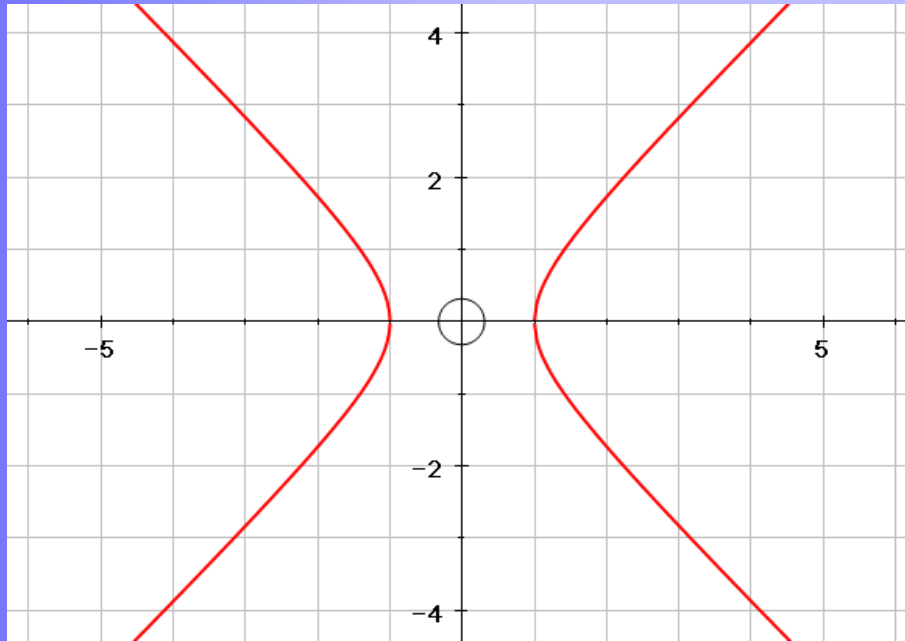
$$x^2 - y^2 = k$$

What are the asymptotes of this graph?

In a rectangular hyperbola, the asymptotes always cross each other at right angles.

# Hyperbola (non-rectangular)

The following equation represents a hyperbola with centre at the origin and cutting the x-axis at  $a$  and  $-a$ .



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The asymptotes have equations:  $y = \pm \frac{b}{a} x$